

Volatility indices and implied uncertainty measures of European government bond futures

This paper shows how to use options to measure forward-looking market uncertainty in European government bond futures.



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Implied volatility and other forward-looking measures of option-implied uncertainty help investors carefully evaluate market sentiment and expectations. We construct several measures of implied uncertainty in European government bond futures. In the first part, we create new volatility indices, which reflect market pricing of subsequently realised volatility of underlying bond futures. We express volatility indices in both price and basis points, the latter being more intuitive to interpret; we document their empirical properties, and discuss their possible applications. In the second part, we fit the volatility smile using the SABR model, and recover option-implied probability distribution of possible outcomes of bond futures prices. We analyse shapes of the implied distribution, track its quantiles over time, calculate its skewness and kurtosis, and infer probabilities of a given upside or downside move in the price of bond futures or in the yield of their underlying CTD bond. We illustrate these complementary measures throughout the note using Bund futures as an example, and show the results for Schatz, Bobl, OAT, and BTP futures in the annex. Such forward-looking measures help market participants quantify the degree of future market uncertainty and thoroughly assess what risks are priced in.

Keywords: bond futures, market expectations, options, probability density function, SABR, VIX, volatility index.

JEL codes: C13, G13, G14, G17

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Volatility indices and implied uncertainty measures of European government bond futures

Jaroslav Baran¹, Jan Voříšek²

Abstract

Implied volatility and other forward-looking measures of option-implied uncertainty help investors carefully evaluate market sentiment and expectations. We construct several measures of implied uncertainty in European government bond futures. In the first part, we create new volatility indices, which reflect market pricing of subsequently realised volatility of underlying bond futures. We express volatility indices in both price and basis points, the latter being more intuitive to interpret; we document their empirical properties, and discuss their possible applications. In the second part, we fit the volatility smile using the SABR model, and recover option-implied probability distribution of possible outcomes of bond futures prices. We analyse shapes of the implied distribution, track its quantiles over time, calculate its skewness and kurtosis, and infer probabilities of a given upside or downside move in the price of bond futures or in the yield of their underlying CTD bond. We illustrate these complementary measures throughout the note using Bund futures as an example, and show the results for Schatz, Bobl, OAT, and BTP futures in the annex. Such forward-looking measures help market participants quantify the degree of future market uncertainty and thoroughly assess what risks are priced in.

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1. Introduction

Option prices offer useful insights into risks surrounding market expectations. If one can recover the probability distribution of underlying asset prices in the future, one can extract a whole set of additional market-implied information about future prices. This is our goal; we use option prices to construct forward-looking measures of uncertainty around the prices of underlying European government bond futures. There are two parts of the note. In the first part, we construct new volatility indices from option prices on German, French, and Italian government bond futures. In the second part, we expand the analysis by recovering the probability distribution of possible outcomes of bond futures prices, implied from options. We use it to estimate complementary measures of implied uncertainty.

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Volatility indices, built upon the methodology of Cboe VIX (2019), have become popular measures of market uncertainty over the short term, across a range of underlying asset classes. They are easily interpretable as they reflect market pricing of subsequently realised volatility, implied from option prices, usually over the next 30 days. Fixed income option-implied volatility indices and products linked to them are already available in the US (for US Treasury futures and interest rate swaps - Cboe TYVIX/SRVIX indices, ICE BofAML MOVE/SMOVE index) and Japan (for JGB futures - S&P/JPX JGB VIX). In the case of other asset classes, the development of volatility indices has a longer history with a number of existing indices across equities, credit, commodities or FX. For a recent overview of available volatility indices across asset classes, we refer to Siriopoulos and Fassas (2019). Other option-implied indicators to measure the uncertainty and sentiment in the markets also have a long history for asset classes such as equities, commodities, or FX.

In the euro area fixed income market, euro swaptions are usually used to monitor the implied volatility of the underlying euro swap rates. There exists a euro version of ICE BofAML SMOVE index, which calculates the weighted average of normalized swaption implied volatilities on 2-year, 5-year, 10-year and 30-year euro interest rate swaps. However, we are not aware of any volatility index on European government bond futures. Such indices would provide insights into investor sentiment and forward-looking market uncertainty of the underlying European sovereign bond market.

We expand the family of volatility indices and propose the construction of new implied volatility indices from options on European government bond futures traded on the Eurex exchange. We follow Cboe TYVIX (2018) methodology with a small adjustment. The underlying instruments for the quoted options are German, French and Italian government bond futures. In the case of Germany, we calculate volatility indices for different maturities from options on Schatz (2-year), Bobl (5-year), and Bund futures (10-year), the go-to reference for euro area yields, and the most traded interest rate derivative on Eurex exchange³.

After revisiting the index construction method, we analyse the historical behaviour of new volatility indices. In the case of the volatility index of Bund futures, we provide a historical comparison with US Treasury volatility, and investigate if the implied volatility index carries some informative value regarding subsequently realised volatility.

In the second part, we expand the analysis by examining the implied probability distribution of possible outcomes of bond futures prices. Recovering implied probability distribution from option prices has become a popular way to assess future market uncertainty, particularly among central banks. In some cases, academic research has been applied in practical tools for market monitoring. For example, the Bank of England publishes probability density functions of future outcomes of the UK stock market and short-term interest rate indices⁴, implied from option prices. The Federal Reserve Bank of Atlanta released a tool that estimates probability distribution and the implied future path of

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https://www.eurexchange.com/resource/blob/1785096/fa2a01aaea3eef5192349ee3ac23c182/data/monthlystat_202002.pdf

⁴ <https://www.bankofengland.co.uk/statistics/option-implied-probability-density-functions>

the three-month average Fed Funds rate⁵, calculated from options on three-month Eurodollar futures and from forward starting three-month USD LIBOR/Fed Funds basis swap spreads. The Federal Reserve Bank of Minneapolis produces on a weekly basis historical option-implied probabilities of a large increase and decrease of the price of the underlying asset, for several asset classes⁶. Central banks' research boasts a long history of investigating methods to extract option-implied probability distributions of underlying asset prices in the future. A study of Clews et al. (2000) is very informative in this regard.

Our objective is to apply a practical method to estimate a well-behaved probability distribution of underlying European bond futures, which we use to compute complementary forward-looking measures of market uncertainty. We achieve this by fitting a SABR model to quoted volatility smile, from which we produce reliable and interpretable distributions, while respecting market data. Once we estimate the implied distribution, we analyse its shape, track quantiles of bond futures' distributions over time, and infer probabilities of specific prices or price ranges of bond futures at different option expiration dates in the future. We show how to track market-implied uncertainty historically, and into the future, in a consistent way.

We complement the theory with several examples to illustrate how such analyses can be used to thoroughly assess what is priced in, in addition to using market forward prices or analysts' estimates. In the Annex, we show the results of presented uncertainty measures for Schatz, Bobl, OAT and BTP futures. Our aim is to provide the reader with a practical and comprehensive approach to estimate implied uncertainty in European government bond futures.

2. Calculation of volatility indices

Implied volatility indices are constructed from out-of-the-money (OTM) call and put option premia. One of the reasons is that the market activity in OTM options is greater than in in-the-money (ITM) options, because they have lower delta, and are thus cheaper to hedge and offer higher leverage. An interest-rate volatility index offers easily interpretable market expectations of interest rate uncertainty, which is often tied to the future outcome of important market drivers, such as upcoming central bank decisions, political events, economic data, etc. The calculation of VIX-type volatility indices across asset classes is based on variance swap pricing. A variance swap exchanges realised (historical) variance against the agreed "variance swap" rate, which reflects the market implied variance. The idea behind is that the price of a variance swap is obtained by replicating its payoff with a portfolio of a discrete set of put and call options, and the underlying futures. For the intuition behind, see for example, Demeterfi et al. (1999) or Bossu (2006), where the authors show that such replication can be achieved by the square of a VIX-type volatility index formula; it is an approximation of the expectation of the annualised variance of returns of the underlying asset over 30 days. We follow the same approach with a small modification.

⁵ <https://www.frbatlanta.org/cenfis/market-probability-tracker.aspx>

⁶ <https://www.minneapolisfed.org/banking/current-and-historical-market--based-probabilities>

The pricing of a variance swap is based on creating a contract with a payoff equal to the variance of returns of the underlying asset, and replicating such a contract with a portfolio of traded put and call options. The cost of this replication strategy would then determine the price of the variance swap. Demeterfi et al. (1999) show that options' portfolio with weights inversely proportional to the square of strike gives an exposure to constant variance, independent of the price of the underlying.

The authors show that the expectation of the average realised variance of the returns of the underlying asset from 0 to T can be replicated by

$$E(\sigma^2) = \frac{2}{T} e^{rT} \Pi - \frac{2}{T} \left(\frac{F}{S^*} - 1 - \ln \left(\frac{F}{S^*} \right) \right) \quad (1)$$

where $\Pi = \int_{K=0}^{S^*} \frac{1}{K^2} p(K) dK + \int_{K=S^*}^{\infty} \frac{1}{K^2} c(K) dK$ is the portfolio of put and call options for continuous strikes K , r is the risk-free rate until option maturity T , F is the forward price of the underlying asset, and S^* is an arbitrary level of strike. In practice, S^* is usually chosen as an approximate ATM forward strike, $S^* = K_{ATM}$. Equation (1) assumes that prices of the underlying asset follow a continuous stochastic process without jumps. VIX-type volatility indices are based on the discretised approximation of (1).

We apply the theory behind the variance swap pricing to calculate implied volatility indices from the options on European government bond futures for monthly expiries traded on Eurex exchange⁷ (Germany 2-year, 5-year, 10-year, France 10-year, and Italy 10-year). We use options on

- Schatz futures
- Bobl futures
- Bund futures
- OAT futures
- BTP futures

These options are available six months ahead, for the next three calendar months, and then for quarterly months of March, June, September, and December. Most of the trading activity is, however, concentrated in maturities below three months, as longer term premia rise with increasing volatility. To be able to track such index consistently over time, it is useful to fix its maturity to a constant, by interpolating between two indices, with closest expiration dates before and after the constant. As in other VIX-type indices, we fix the maturity to a constant one month period, 30 calendar days, i.e. the index measures the annualised implied volatility of underlying futures contracts over one month, expressed in price σ_{price} and basis points σ_{bp} . We focus on the short-term constant 30-day ahead period, due to enhanced liquidity in the front contracts, however, the extension to other periods is straightforward. To get to 30 days, we interpolate the calculated variance of the near term and next term option. During most days, the near term option expires in less than 30 days, and next term option between 30 and 60 days. For those days, when the near term option expires in more than 30 days, we extrapolate the 30-day volatility from the near and the next term option. Linear

⁷ <https://www.eurexchange.com/exchange-en/products/int/fix-opt/government-bonds/>. An extension to other euro instruments, for example, options on 3M Euribor or euro swaptions, is straightforward.

interpolation/extrapolation in most cases offers good approximation of 30-day maturity, although the term structure of implied variance is neither constant nor linear. In the case of options on Bund futures, there exist also weekly options with Friday expiry up to first five weeks, which could remove some of the interpolation error, however, the limited activity in weekly options prevents us from using them.

The step-by-step calculation of a fixed income volatility index, based on the square root of annualised variance, is well explained in TYVIX whitepaper, Cboe TYVIX (2018). We follow the same theoretical approach, using the discrete approximation of variance fair value from (1), with a small modification in the correction term

$$\sigma_{near}^2 = \frac{2}{T} e^{rT} \left(\sum_{i, K_i > K_{ATM}} \frac{\Delta K_i}{K_i^2} c(K_i) + \sum_{i, K_i < K_{ATM}} \frac{\Delta K_i}{K_i^2} p(K_i) \right) - \frac{2}{T} \left(\frac{F}{K_{ATM}} - 1 - \ln \left(\frac{F}{K_{ATM}} \right) \right) \quad (2)$$

where

- $c(K_i)$ and $p(K_i)$ are settlement prices of call and put options with strike K_i . Call options with strikes $K_i > K_{ATM}$, and put options with strikes $K_i < K_{ATM}$ are included in the calculation. For $K_i = K_{ATM}$ we take the average price of call and put $\frac{c(K_i)+p(K_i)}{2}$. K_{ATM} is the closest to at-the-money strike which separates puts and calls used in the calculation.
- ΔK_i is the adjusted step between the strikes defined as half the distance between higher and lower strike from K_i , $\Delta K_i = \frac{K_{i+1}-K_{i-1}}{2}$ for $i=2, \dots, n-1$, as some strikes may be missing due to missing quotes. For the lowest and the highest strike used, $\Delta K_1 = K_2 - K_1$ and $\Delta K_n = K_n - K_{n-1}$.
- $\frac{F}{K_{ATM}} - 1 - \ln \left(\frac{F}{K_{ATM}} \right)$ is the adjustment factor which substitutes for the missing ATM strike, with F being the price of the underlying futures.
- e^{rT} returns the forward value with interest rate r interpolated from repo rates of STOXX® GC Pooling repo rate indices with maturity being the expiration date of the option⁸, our proxy for risk-free rate.
- σ_{near}^2 is the price variance of the underlying futures contracts implied from prices of listed near term options with expiration T .

Equation (2) is the discretised version of (1). The calculation returns the weighted sum of daily settlement prices of OTM call and put options on bond futures, with weights inversely proportional to the square of strike. Such weights ensure that the sensitivity of an option portfolio is not affected by changes in options' implied volatilities (options' portfolio vega stays constant, and volatility exposure does not need to be regularly rebalanced if the price/implied volatility of options changes). We exclude options with prices of 0.01, the minimum price quotation, in case of Bund, OAT and BTP options, and 0.005 in case of Schatz and Bobl options. We include all the other quoted options for all available strikes in order not to lose any valuable information that could otherwise lead to an underestimation of the implied volatility⁹. There are always at least two call and two put options used in calculation.

⁸ <https://www.stoxx.com/gc-pooling>

⁹ We also include less liquid OTM options in the calculation which, in practice, makes the index harder to replicate with a series of underlying options. If products on such index were to be created, more calibration

The difference between the VIX formula and the equation (2) is the adjustment factor on the right-hand side of (2). The log function in (2) can be approximated with Taylor series expansion, for example, with the first two terms

$$\ln\left(\frac{F}{K_{ATM}}\right) \approx \left(\frac{F}{K_{ATM}} - 1\right) - \frac{1}{2}\left(\frac{F}{K_{ATM}} - 1\right)^2.$$

Then the adjustment factor is approximated by

$$\frac{2}{T}\left(\frac{F}{K_{ATM}} - 1 - \ln\left(\frac{F}{K_{ATM}}\right)\right) \approx \frac{1}{T}\left(\frac{F}{K_{ATM}} - 1\right)^2, \quad (3)$$

which is the term used in the VIX formula. Although the size of the error would in most cases be negligible, there is no practical need to introduce another approximation into the formula, as noted in Pimbley and Phillips (2018).

The calculation gives the variance of the returns of an underlying bond futures contract until maturity of the option. We follow the same approach as for the other VIX-type indices

- at-the-money strike K_{ATM} is the strike which gives the lowest absolute difference between the prices of all calls and puts,
- the calculation includes all puts with strikes lower than ATM strike and all calls with strikes higher than ATM strike.

One-month implied price volatility is then calculated by interpolating between two listed options with closest expiration dates, for which the data are available (near term σ_{near}^2 and next term σ_{next}^2 variances¹⁰),

$$\sigma_{price} = 100 \times \sqrt{\frac{365}{30}(wT_{near}\sigma_{near}^2 + (1-w)T_{next}\sigma_{next}^2)} \quad (4)$$

where weight $w = \frac{T_{next} - \frac{30}{365}}{T_{next} - T_{near}}$, and T is option's time to expiration expressed as a year fraction.

To calculate volatility in basis points, we use bond price-yield sensitivity relationship. We multiply the price volatility of the near and the next term option (expected % change in the price of the underlying

thought should be given to the exclusion rules to balance the information content of the index, with hedging practicability and resilience against market price manipulation. When preparing the data and dealing with illiquidity, one could also consider including only options with some traded volume and open interest, however, this could substantially reduce the available dataset. Another method on how to identify and remove illiquid deep OTM options could be to calculate options' vega or delta and remove quotes with negligibly small sensitivities.

¹⁰ For example, in the case of volatility index on Bund futures, there are on average 9 call and 9 put options used in calculation of the near term variance and 15 calls and 16 puts used in calculation of next term variance, see the Annex.

Bund futures) by 100 and futures price F and divide it by the DV01 of the underlying futures ($DV01_{futures} = \frac{DV01_{CTD}}{\text{conversion factor}}$, where CTD is the cheapest-to-deliver bond)

$$\sigma_{near\ bp}^2 = \sigma_{near}^2 \times \left(\frac{100 \times F_{near}}{DV01_{futures\ near}} \right)^2, \quad \sigma_{next\ bp}^2 = \sigma_{next}^2 \times \left(\frac{100 \times F_{next}}{DV01_{futures\ next}} \right)^2, \quad (5)$$

Finally, we calculate the basis point implied volatility index as the square root of interpolated basis points variances between near and next term options

$$\sigma_{bp} = 100 \times \sqrt{\frac{365}{30} (wT_{near}\sigma_{near\ bp}^2 + (1-w)T_{next}\sigma_{next\ bp}^2)} \quad (6)$$

where $\sigma_{near\ bp}^2$ is the yield implied variance of the near term option. σ_{price} measures the annualised price volatility of underlying bond futures over the next 30 days, while σ_{bp} is the annualised implied basis points volatility.

3. Empirical analysis and applications of volatility indices

The following graph shows the implied volatility index in case of Bund futures: the *Bund volatility index*, based on (6), in basis points annualised. Selected volatility spikes are briefly discussed later.

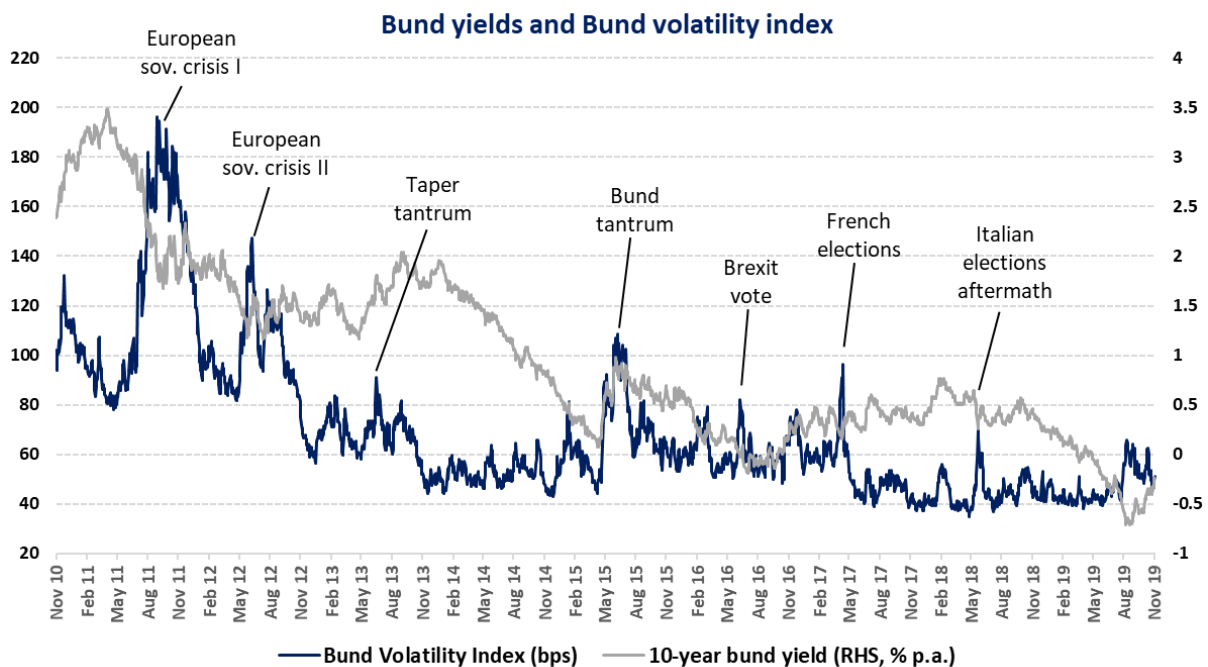


Figure 1: Bund Volatility Index in basis points with highlighted volatility spikes. *Source: Bloomberg, authors' calculations.*

In the case of equities, empirical evidence suggests that market downturn is almost always associated with heightened volatility, however, fixed income volatility appears to be less directional; implied volatility increases both during a Bund sell-off and a Bund rally, given its flight to safety status.

We can observe several other characteristics, similar to volatility of other asset classes. For example, Bund implied volatility suggests regime-like behaviour: average Bund implied volatility appears to have moved from higher values in 2011-2012 to lower values in 2013-2014, then the average volatility increased between 2015 and the beginning of 2017 before decreasing again. We can also observe mean reversion; every sudden spike eventually returned towards calmer periods, occasionally establishing a new regime. Volatility spikes and drops often reflect pricing of the uncertainty of a particular event: the volatility increases as the event date approaches, and drops after the event passed. In this section, we look at the history of Bund volatility index, compare it with the volatility index of US treasuries (TYVIX), and with subsequently realised volatility of bund returns. Then we examine the correlations among the newly constructed volatility indices and their underlying bond futures. Our aim is to describe some of the characteristics of new volatility indices and discuss their possible use.

3.1. Comparing Bund implied volatility with US Treasury implied volatility

The Bund volatility index has been moving together with the US Treasury implied volatility index, TYVIX, with a few exceptions related to European events, such as developments in the euro area sovereign credit spreads or repricing of the ECB rate expectations.

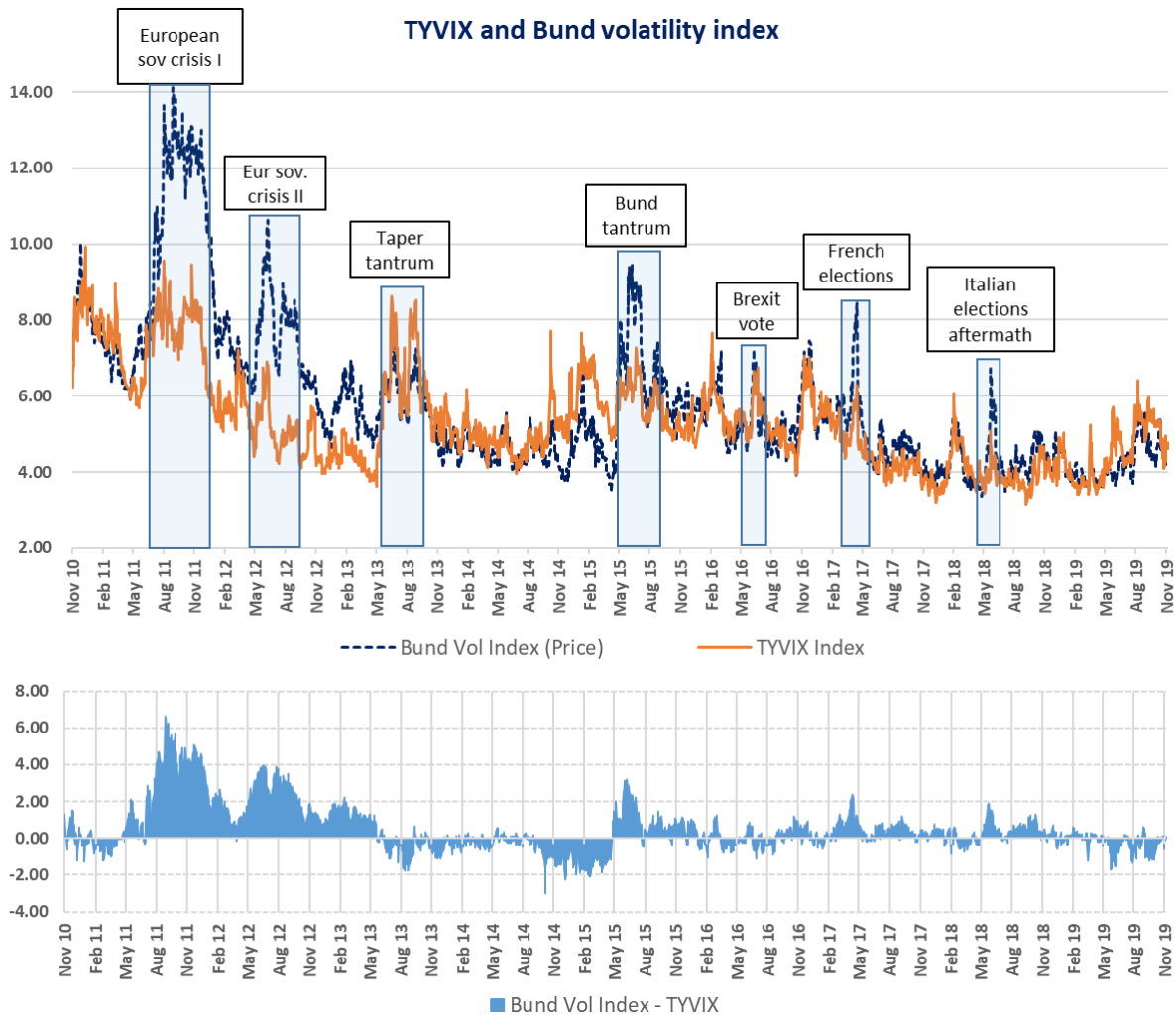


Figure 2: TYVIX and Bund volatility index, expressed in price percentage terms, annualised. *Source: Bloomberg, authors' calculations.*

July 2011 - December 2011

The euro area sovereign debt crisis is fully manifested and volatility spiked to levels not seen since 2008/2009 crisis. Concerns around euro area sovereign defaults and recession fears drove risky asset prices lower, credit spreads wider, and bond yields, even the highly rated ones, higher. Over the course of the months, periods of pessimism had been alternating with periods of optimism on hopes that central banks and government would tackle the crisis. Bund implied volatility climbed well above US Treasury implied volatility for a prolonged time. Coordinated central bank action to provide liquidity in major currencies at the end of November 2011 reduced market tensions and fuelled a subsequent rally.

May-June 2012

Bund yields declined considerably as demand for safety increased on renewed concerns about the euro area debt crisis and an increased perception of sovereign default risk. At the same time, the euro area growth outlook deteriorated and ECB rate hike expectations were pushed back. Implied volatility in Bund futures recorded almost a twofold increase while spillover to US Treasury volatility was limited, suggesting elevated rates uncertainty was concentrated mainly in the euro area. Volatilities decreased substantially following ECB easing measures.

May-June 2013

Implied volatilities surged in both Bund and Treasury futures, accompanied by the increase in US Treasury and global yields, which reacted strongly to the Fed announcement about future reduction in the pace of Treasury bonds purchases ("taper tantrum").

April-June 2015

Following the announcement of the ECB asset purchase programme, Bunds reached unprecedentedly high valuations with record low yields, and exceptionally flat curves. Bund yields reached single digit basis points before climbing to almost 1% within less than two months. Bunds sold off sharply, curves re-steepened, and both implied and realised volatilities spiked. Higher oil prices, which lifted market inflation expectations, may have contributed to the sell-off. Some anecdotal evidence suggested that the jump in volatility forced volatility sensitive market participants to unwind their positions, prompting further sell-off. The move was amplified by the reported lack of liquidity. Bund implied volatility notably surpassed its US counterpart.

June 2016

Volatility spiked the day after the UK referendum results unexpectedly showed that the majority voted to leave the EU, triggering flight to quality flows. The spike was short lived, volatility fell and the drop in yields partly reversed.

April 2017

Heightened political uncertainty in the run-up to the first round of French presidential elections resulted in an implied volatility spike and higher Bund valuations. Polls suggested that all top four candidates, two of them less-centrist, had sizeable chances of succeeding into the second round. In the run-off, voters could have possibly faced a choice between an anti-euro far-right and a far-left candidate. The result of the first round ruled out this option, triggering a risk rally, and leading to a drop in implied volatilities as perceived political risk in Europe has substantially receded.

May-June 2018

The new Italian coalition government budget plans and anti-euro rhetoric led to increased investor perception of political risk and loss of confidence. The proposed spending plans were seen as unlikely to meet Europe's fiscal rules, and potentially increasing the country's already high debt levels, putting its debt sustainability at risk. Italian yields jumped, risk assets fell, and Bund yields fell on flight to quality. Realised volatility spiked and implied volatility almost doubled within a month. Although Italian bonds struggled to stabilise and both political and market risk premia remained elevated, Bund implied volatility receded in June towards its average levels.

Rolling correlation of daily changes between BVI and TVIX indices shows a positive but changing relationship between the two, with low correlation indicating occurrence of events which were more specific to either Bunds or US Treasuries.

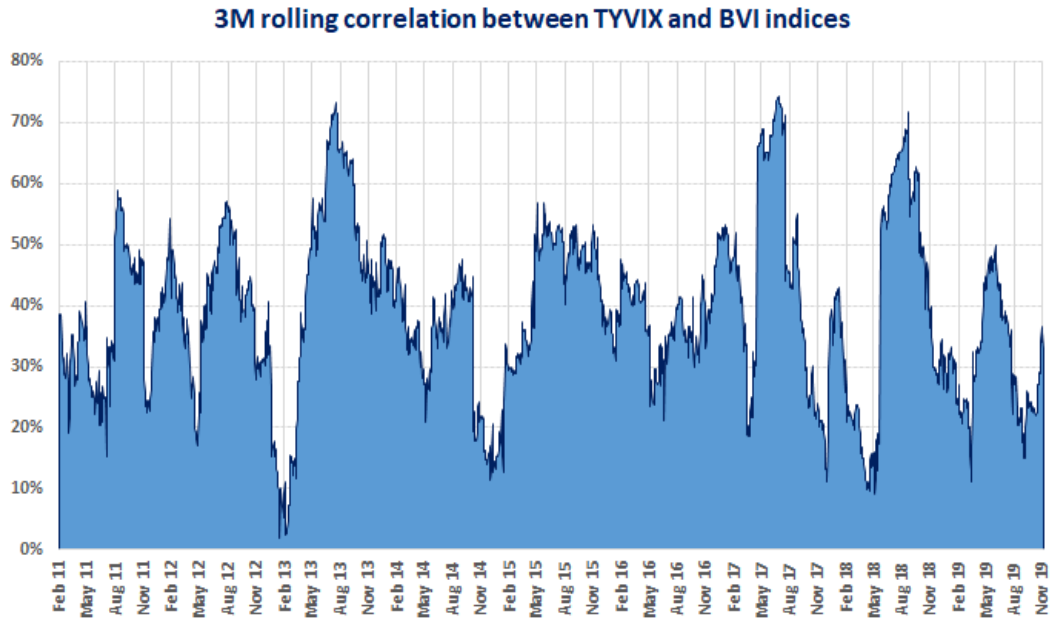


Figure 3: 3-month rolling correlation between BVI and TYVIX indices. *Source: Bloomberg, authors' calculations.*

Bund Volatility Index and TYVIX histograms

We construct histograms of BVI and TYVIX index values to examine their shapes and describe their statistical properties.

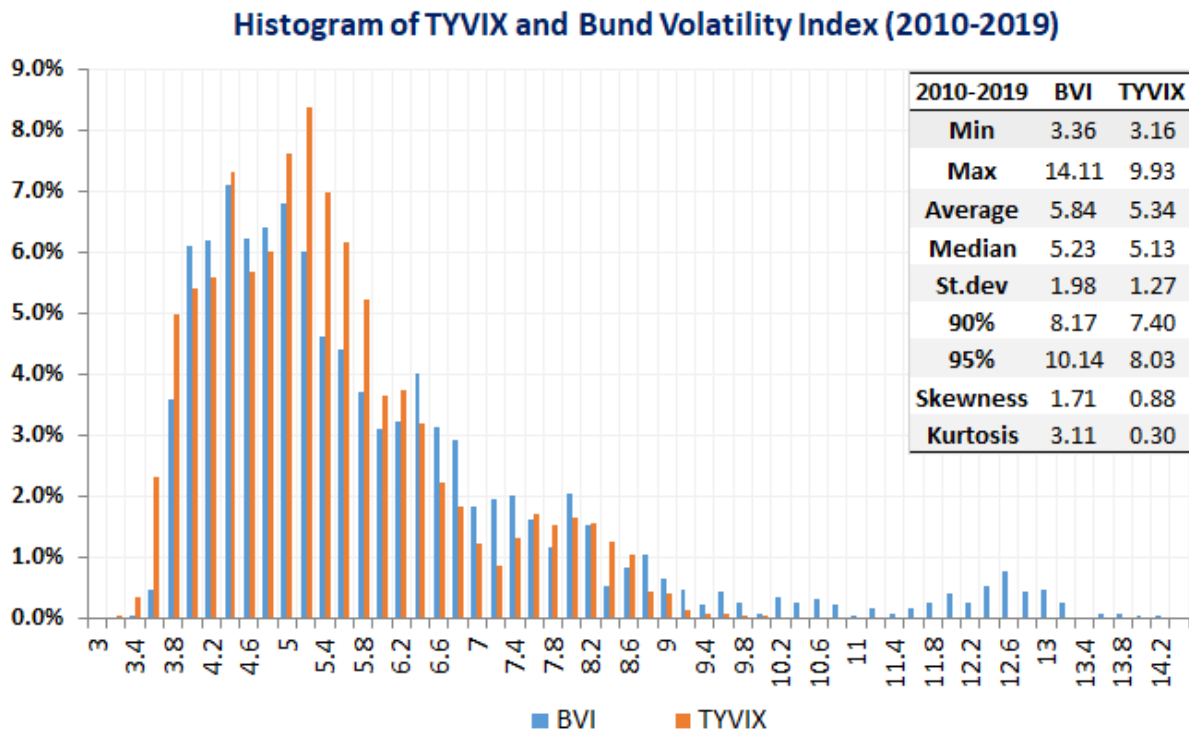


Figure 4: Full sample (Nov 2010 - Nov 2019) histograms of BVI and TYVIX index values shows longer and heavier tails for BVI. *Source: Bloomberg, authors' calculations.*

The European debt crisis largely contributed to higher Bund volatility and affected the shape of its distribution in our full sample. While both BVI and TYVIX are positively skewed with a long right tail, Bund volatility skewness is higher, exhibiting a longer and heavier tail. A bigger mass of the distribution of Bund volatility is concentrated in the right tail, as indicated by higher values of standard deviation, skewness, and kurtosis. Higher values for Bund volatility are evident for each chosen descriptive statistic in the table; however, the higher overall level of Bund price volatility relative to US Treasuries is also affected by higher sensitivity of Bund futures contracts to interest rate changes.

Such direct comparison is somewhat misleading because the two futures contracts do not bear the same market risk. In fact, the DV01 of one Bund futures contract has been increasing relative to the DV01 of Treasury futures contract over the last decade as Bund yields have been falling more, relative to US Treasury yields, e.g. a one basis point move in CTD yield corresponds to around 30% bigger move in the Bund futures price than the Treasury futures price.

To make the indices comparable, we adjust them for the DV01 difference and express the TYVIX index in basis points using (5). We can observe that US Treasury implied volatility in basis points is on average higher than Bund implied volatility, even when we take into account the exceptional period of the European sovereign debt crisis. While the implied volatility of US Treasury yields has been on average higher than implied volatility of Bund yields, the Bund yield volatility has exhibited a higher degree of variability in the full sample.

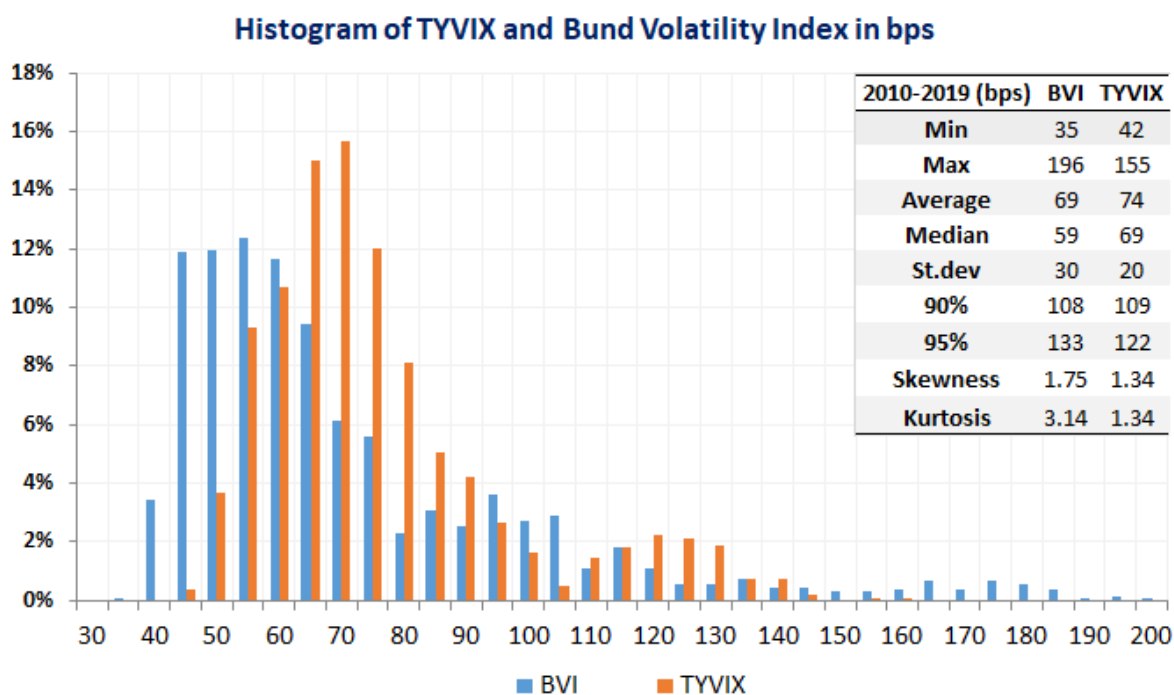


Figure 5: Full sample (Nov 2010 - Nov 2019) histograms of BVI and TYVIX index values, expressed in basis points p.a.. *Source: Bloomberg, authors' calculations.*

3.2. Testing the direction of future volatility with volatility index

We compare the *Bund volatility index* with subsequently realised volatility to see if the options market correctly predicts the direction of future volatility. First, we compare realised volatility over the last 30 calendar days with the current value of the implied volatility index.

- If today’s index value is higher (lower) than realised volatility, we test if the subsequently realised volatility over the next 30 days will also be higher (lower), as options currently price.

A similar study on options on Bund futures was carried out by Neuhaus (1995). Using the Bund volatility index on a sample of data from November 2010 to August 2019, we can observe that the index correctly predicts the direction of subsequently realised volatility by 63%. Interestingly, periods of lower realised volatility are predicted almost with certainty (if the volatility index value is lower than realised volatility over the last 30 days, then subsequently realised volatility will also be lower than the realised volatility over the last 30 days, with the likelihood of 93%). Implied volatility is likely to decrease if no important market-driving event is scheduled until the option's expiration date, and realised volatility follows. Periods of higher realised volatility are predicted less accurately, at 56%. One possible explanation could be the existence of an implied volatility risk premium.

Prediction/ Outcome	Higher	Lower	Total
Higher	56%	8%	63%
Lower	44%	92%	37%

Table 1: Prediction accuracy for the direction of subsequently realised volatility. *Source: Bloomberg, authors’ calculations.*

The following graph displays the *Bund volatility index* together with subsequently realised volatility (annualised standard deviation of logarithmic Bund futures returns¹¹ subsequently realised over the next 30 days).

¹¹ To achieve a continuous series of percentage returns, we form a continuous futures contract using proportional adjustment, i.e. we take the ratio of the old price to new price to smooth historical prices and avoid jumps at roll dates.

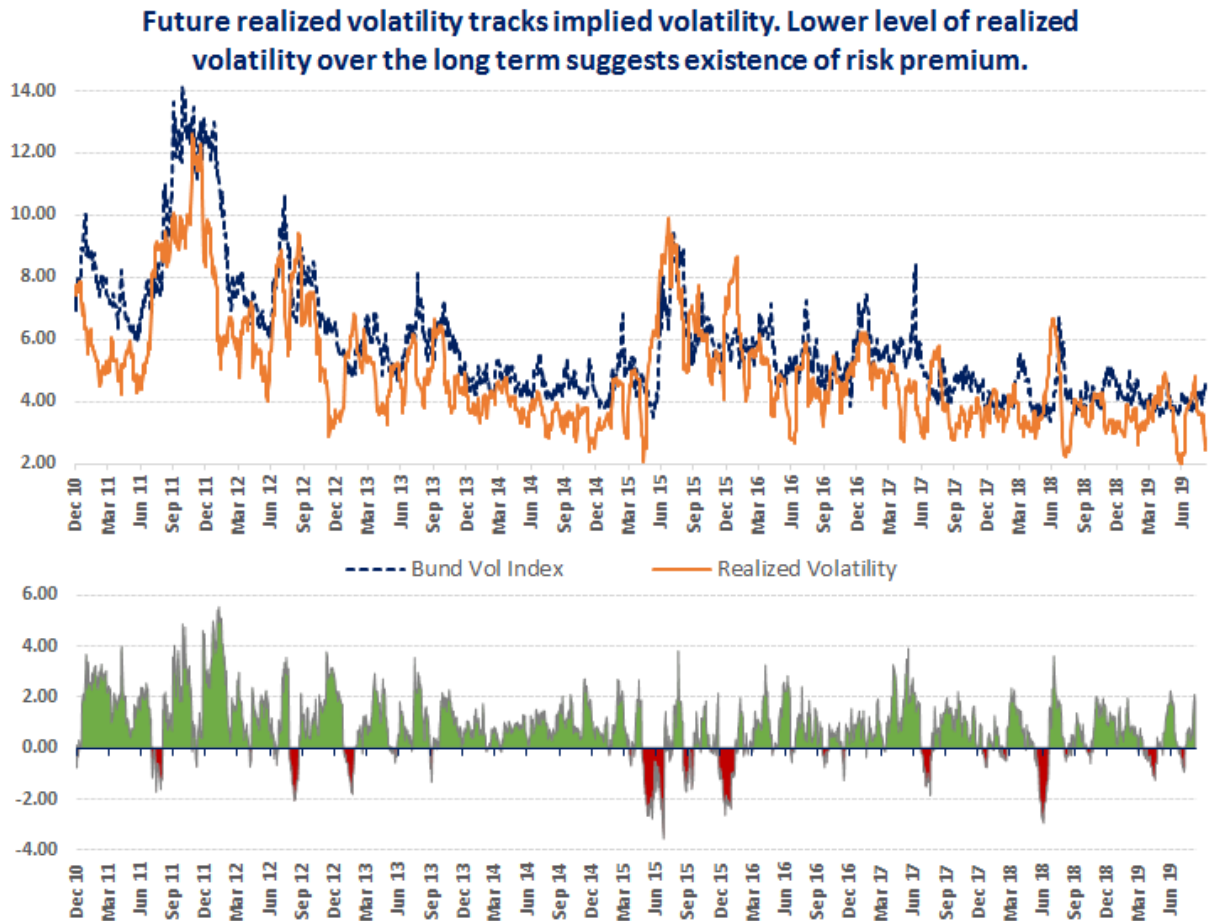


Figure 6: Bund Volatility Price Index compared with subsequently realized volatility of Bund futures.
Source: Bloomberg, authors' calculations.

With several exceptions, the realised volatility has been most of the time below the Bund implied volatility index. This suggests the possible existence of a *varying* volatility risk premium, which might reflect the compensation for the risk of unexpected spikes in realised volatility.

3.3. Correlation of volatility indices

Using recent 1-year data, from November 2018 to November 2019, we construct a cross-asset correlation matrix to show the relationship among volatility indices and their underlying bond futures.

1-year correlation	Schatz vol	Obl vol	Bund vol	OAT vol	BTP vol	Schatz	Obl	Bund	OAT	BTP
Schatz vol	1									
Obl vol	0.44	1								
Bund vol	0.36	0.88	1							
OAT vol	0.28	0.76	0.83	1						
BTP vol	0.17	0.28	0.29	0.29	1					
Schatz	0.41	0.33	0.22	0.20	0.15	1				
Obl	0.36	0.22	0.08	0.08	0.07	0.88	1			
Bund	0.27	0.11	-0.04	-0.02	0.00	0.69	0.92	1		
OAT	0.24	0.04	-0.12	-0.09	-0.10	0.62	0.84	0.94	1	
BTP	0.02	-0.07	-0.18	-0.16	-0.44	0.10	0.23	0.32	0.49	1

Table 2: Correlation of daily changes in volatility indices and returns on their underlying bond futures. Futures prices were adjusted proportionally, to achieve a continuous return time series. *Source: Bloomberg, authors' calculations.*

We can observe a negative correlation between the returns on Italian bond futures and their implied volatility, suggesting that an increase in Italian yields is accompanied by higher expected yield uncertainty in the future. Italian bond futures are negatively correlated to other volatility indices, signalling their risk-off nature; they are likely to underperform when risk sentiment goes sour, which is often accompanied by increased volatility across countries. Positive correlation among volatility indices indicates that implied volatilities rise and fall together across countries. On the other hand, over the last year, Bund futures returns have been almost uncorrelated with Bund futures implied volatility, which is less directional; it increases during both a Bund rally and a sell-off. Including an uncorrelated volatility index into the portfolio can offer diversification benefits and decrease the overall portfolio volatility.

3.4. Use of fixed income volatility indices

Volatility indices of bond futures, due to their empirical properties, have some desirable features for portfolio construction and optimization, or market monitoring. Similarly to volatilities of other assets, fixed income volatility exhibits clustering, mean reversion, and regime-like behaviour. The distribution is usually positively skewed, i.e. periods of calm are observed frequently, until sudden less-frequent spikes occur, which could leave the volatility elevated before eventually returning to a period of calm. These characteristics offer interesting hedging or portfolio diversification benefits when using volatility as an asset class.

For example, when implied volatility increases as prices of its underlying asset fall, buying volatility would hedge the portfolio of underlying assets in case of a downturn. Selling volatility, on the other hand, can be an attractive strategy in case of volatility spikes, due to its mean reverting property and the possible existence of a volatility risk premium. Instruments tracking a bond volatility index could be also interesting for trading the volatility between bond and stock markets. Volatility indices may

also be used to target market timing of asset allocation as their movement has some predictive power for the movement of the underlying¹².

Trading fixed income volatility using options is often done via straddles, i.e. buying or selling at-the-money or out-of-the-money calls and puts. This requires regular recalibration as volatility changes when the underlying asset price changes. Some fixed income investors, however, may not be ready to hedge the volatility of their portfolios by actively managing delta-hedged option straddles, or other similar option strategies, or by using variance or volatility swaps. For many fixed income investors, gaining or reducing volatility exposure directly can be easier via futures or ETF contracts than via fixed income options. Instruments tracking volatility indices could offer a simplified way to gain exposure to volatility only, and do not require regular recalibration as opposed to trading volatility via options.

Potential new market instruments based on European fixed income volatility indices, such as futures, options, ETFs/ETNs or their OTC equivalents, could further encourage use of volatility as an asset class. Such products could potentially increase the liquidity in underlying options and futures markets, which would be used by dealers to hedge the trading flows from new volatility products. One could also think of a composite volatility index, by averaging across different volatility indices, and thus combining maturity and country volatility features into one indicator. This could be used to monitor bond market implied uncertainty across the euro area as a whole.

4. Option implied probability distributions

While implied volatilities provide useful information about future uncertainty, through option premia, we can extract a whole set of additional information about prices of their underlying assets in the future. For example, we can infer probabilities associated with prices of underlying assets at different dates in the future and display them in an easily digestible way through fan charts. We can track certain quantiles of the distribution over time, or measure how a probability of a certain interest rate shock evolves. We can thus analyse the shapes of the entire distributions of the underlying in the future in different ways. This additional information content from option prices can complement a more established analysis of market forward prices and analysts' forward price point estimates.

Option prices reflect the probability of being exercised through their time value and their strike price. This probability evolves over time as prices and the realised volatility of the prices of the underlying asset change. Probability distributions from option prices are often recovered by assuming a specific form of the density function, or a specific stochastic process for the underlying asset, or by differentiating the option pricing function directly. For a relevant overview of the methods which estimate option-implied probability density function (PDF) we refer to Bliss and Panigirtzoglou (2002). We apply the last approach, which is widely used, and allows creating smooth and well-behaved densities. It is based on the results of Breeden and Litzenberger (1978), who show that risk-neutral PDF can be recovered from the second derivative of the call price with respect to strike, discounted to its present value.

¹² Cboe (2016) paper shows that the relative performance between traditional assets (equities, bonds, cash) can be predicted by changing relationship between bond and stock volatility indices.

The price of the call and put option equals the present value of the expected payoff (under the risk-neutral density),

$$\begin{aligned} c_K &= e^{-r(T-t)} E[\max(S_T - K, 0)] \\ p_K &= e^{-r(T-t)} E[\max(K - S_T, 0)] \end{aligned} \quad (7)$$

which can be re-written by integrating the product of density (PDF) and option payoff over bond futures prices and discounting the result

$$\begin{aligned} c(K) &= e^{-r(T-t)} \int_0^\infty \max(S_T - K, 0) f(S_T) dS_T = e^{-r(T-t)} \int_K^\infty (S_T - K) f(S_T) dS_T \\ p(K) &= e^{-r(T-t)} \int_0^K (K - S_T) f(S_T) dS_T \end{aligned} \quad (8)$$

where c and p are call and put option prices at time t , expiring at time T , with strike price K , price of the underlying asset at time T S_T , and a risk-neutral density $f(S_T)$ that specifies the probability of possible outcomes for underlying asset prices.

Its first derivative with respect to the strike price K recovers the cumulative distribution function *CDF*

$$\frac{\partial c}{\partial K} = -e^{-r(T-t)} \int_K^\infty f(S_T) dS_T = -e^{-r(T-t)} (1 - CDF(K)) \quad (9)$$

and its second derivative returns the PDF

$$\frac{\partial^2 c}{\partial K^2} = e^{-r(T-t)} f(K). \quad (10)$$

The density $f(K)$ is estimated by numerically differentiating discrete option prices twice with regards to their strike. It can be approximated from discrete price as

$$f(K) = e^{r(T-t)} \frac{\partial^2 c}{\partial K^2} = e^{r(T-t)} \frac{c(K-\Delta K, T) - 2c(K, T) + c(K+\Delta K, T)}{(\Delta K)^2} \text{ in the limit, for } \Delta K \rightarrow 0. \quad (11)$$

The idea is that the density function of the underlying futures equals the discounted second derivative of the call price with respect to the strike, which can be approximated by going long call options with strikes $K + \Delta K$ and $K - \Delta K$, and going short two call options with strike K (what is known as a butterfly strategy).

Since option quotes are available only for a predefined range of strikes, the PDF retrieved only from quoted prices can become less stable, in particular when options approach expiration. To create a smooth shape of the density, which is easier to interpret, while respecting market pricing, interpolation methods are used to increase the option data granularity within and outside the quoted range. In this process,

- OTM option prices are first transformed into implied volatilities using the Black pricing formula (volatility-strike pairs of OTM calls and OTM puts are created). In our calculations, we excluded options with prices of 0.01 (0.005 in case of Schatz and Bobl options).
- Implied volatilities are interpolated and extrapolated with a convenient interpolation method to produce a smooth volatility smile.
- The sufficiently large number of interpolated volatilities is converted back into smoothed theoretical option prices with Black formula.
- Second derivative of smoothed option prices with respect to strike is calculated numerically to obtain the PDF.

Fitting the volatility smile

The chosen interpolation and extrapolation method should ensure no arbitrage conditions for the resulting option prices, which could lead to unstable density, for example, density with negative values. We examined cubic splines (piecewise cubic polynomial) as in Malz (2014) and quadratic function as in Shimko (1993) or O'Donnell and Keeffe (2016) for the interpolation inside the quoted range. Outside the range of quoted strikes, in the wings, we considered two methods, polynomial extrapolation, and assuming constant implied volatility, as in Malz (2014), however, this sometimes produces unwanted jumps at the bounds (the last available lower and upper strikes), or unrealistic density shapes in the tails that would theoretically lead to arbitrage opportunities (practically, however, there are no quotes).

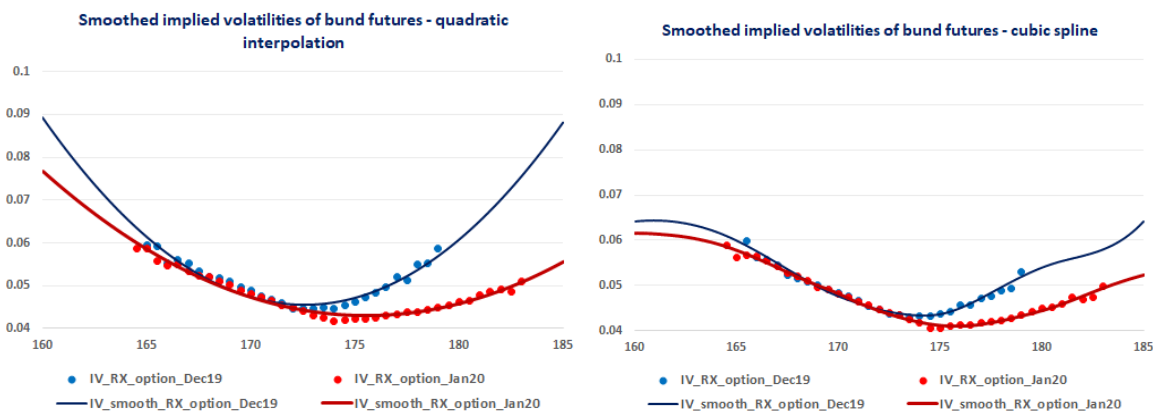


Figure 7: Implied volatilities from options on Bund futures and their quadratic (left) and cubic spline (right) interpolations as of 15th October 2019. Expiries displayed are December 2019 option (RXZ9) with December 2019 Bund futures underlying (RXZ9) and January 2019 option (RXF0) with March 2020 Bund futures underlying (RXH0). *Source: Bloomberg, authors' calculations.*

For the ease of interpretation, we want to produce an arbitrage-free interpolated volatility smile which would be close enough to market implied smile, its shape would correspond to empirically observed shapes of volatility smiles, and it would result in a well-behaved density, in particular, in the tails. The fitting function should therefore generate smooth transition from the central part to the wings.

We overcame the above issues by fitting the volatility smile using the SABR volatility model of Hagan et al. (2002), which has proven to capture the smile and extrapolate beyond the quoted range well, also on days of unexpected volatility spikes. In SABR model, the volatility is given by

$$\sigma(K, F) = \frac{\alpha}{(FK)^{\frac{(1-\beta)}{2}} \left(1 + \frac{(1-\beta)^2}{24} \log^2 \frac{F}{K} + \frac{(1-\beta)^4}{1920} \log^4 \frac{F}{K} \right)} \times \frac{z}{x(z)} \times \left(1 + \left(\frac{(1-\beta)^2}{24} \frac{\alpha^2}{(FK)^{1-\beta}} + \frac{1}{4} \frac{\rho\beta v \alpha}{(FK)^{(1-\beta)/2}} + \frac{2-3\rho^2}{24} v^2 \right) T \right),$$

$$\text{where } z = \frac{v}{\alpha} (FK)^{(1-\beta)/2} \log \frac{F}{K} \text{ and } x(z) = \log \left(\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho} \right). \quad (12)$$

Parameter α represents the magnitude of the level of ATM implied volatility and shifts the overall volatility smile upwards or downwards, β describes the dynamics of the process of futures price and affects the skew, similarly to ρ , where ρ is the correlation between changes in the futures price and its volatility, and affects the slope of the skew, and v is the volatility of volatility of futures price, which controls the curvature of the smile.

Given that ρ parameter produces a similar effect on smile as β , to make parameters more stable, we fixed β to 1 (assuming lognormal distribution for futures prices F). We calibrated other parameters by minimising errors between observed market volatility and SABR model volatility in (12). Modelled $\sigma(K, F)$ was then used in Black formula to obtain theoretical option price.

The table below summarizes the average parameters of the modelled near and next term volatilities for each bond futures contract, calibrated daily on the sample from June 2018 to November 2019, with fixed $\beta=1$.

Contract	Near term			Next term		
	α	ρ	v	α	ρ	v
Schatz	0.01	0.38	2.39	0.01	0.35	1.89
Obl	0.02	0.04	2.71	0.02	0.06	1.82
Bund	0.05	-0.05	3.08	0.04	-0.22	1.44
OAT	0.04	-0.21	2.12	0.04	-0.22	1.38
BTP	0.10	-0.49	2.93	0.09	-0.53	1.53

Table 3: Average values of SABR model parameters, given $\beta=1$ for each contract. *Source: Authors' calculations.*

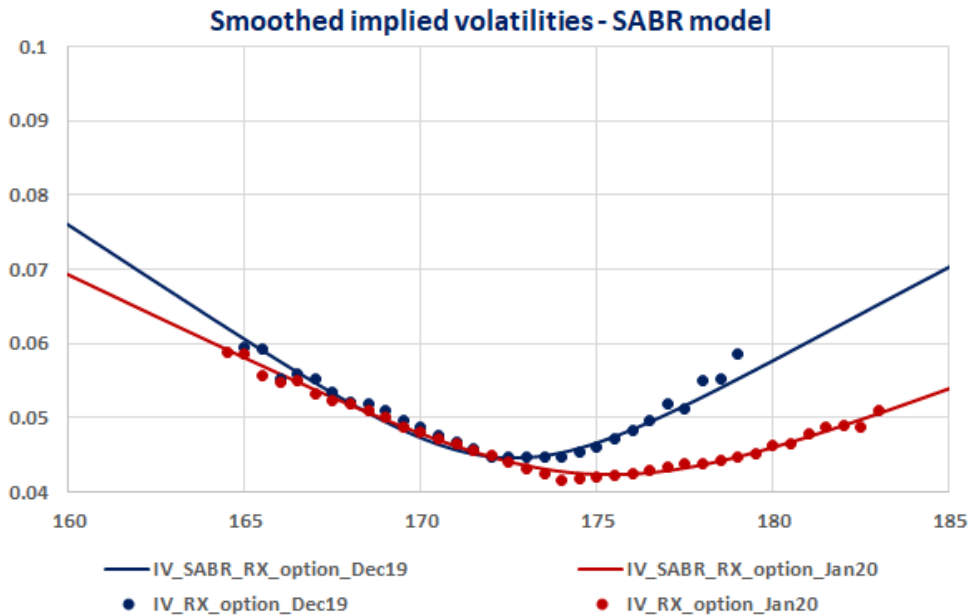


Figure 8: Implied volatilities from options on Bund futures and their SABR modelled values as of 15 October 2019. Expiries displayed are December 2019 option (RXZ9) with December 2019 Bund futures underlying (RXZ9) and January 2019 option (RXF0) with March 2020 Bund futures underlying (RXH0). *Source: Bloomberg, authors' calculations.*

In the remainder of the paper, we use the SABR model to fit implied volatility smile with sufficient granularity between strike prices. Once we convert a granular grid of smoothed implied volatilities into smoothed option prices, we numerically differentiate them with respect to their strike prices to obtain their implied densities.

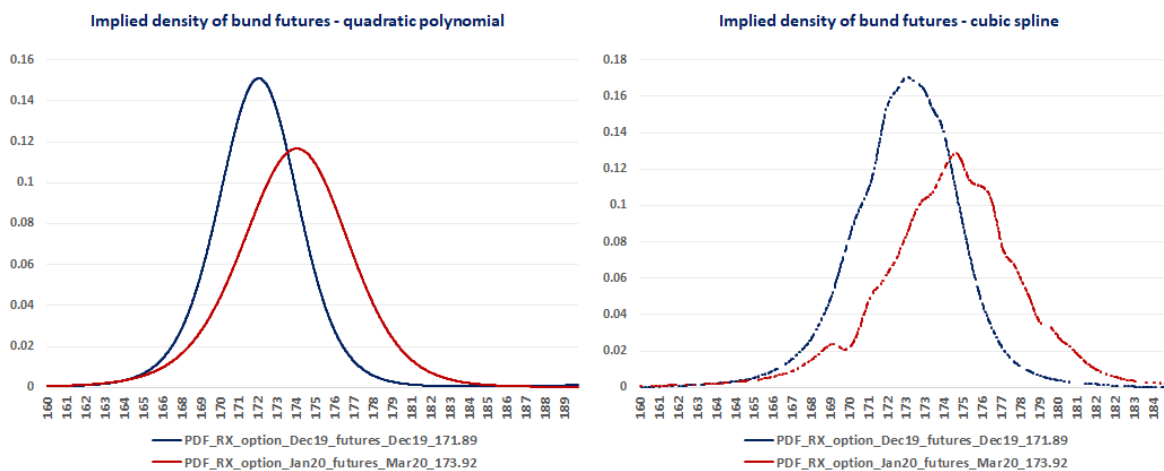


Figure 9: Implied density from two option contracts on Bund futures as of 15 October 2019 (December 2019 option (RXZ9) with December 2019 Bund futures underlying (RXZ9) and January 2019 option (RXF0) with March 2020 Bund futures underlying (RXH0)). The uncertainty about Bund futures outcomes declines as option maturity decreases (December expiry shows shorter tails and lower variance). The density on the left is retrieved using quadratic interpolation, the density on the right via cubic spline. *Source: Bloomberg, authors' calculations.*

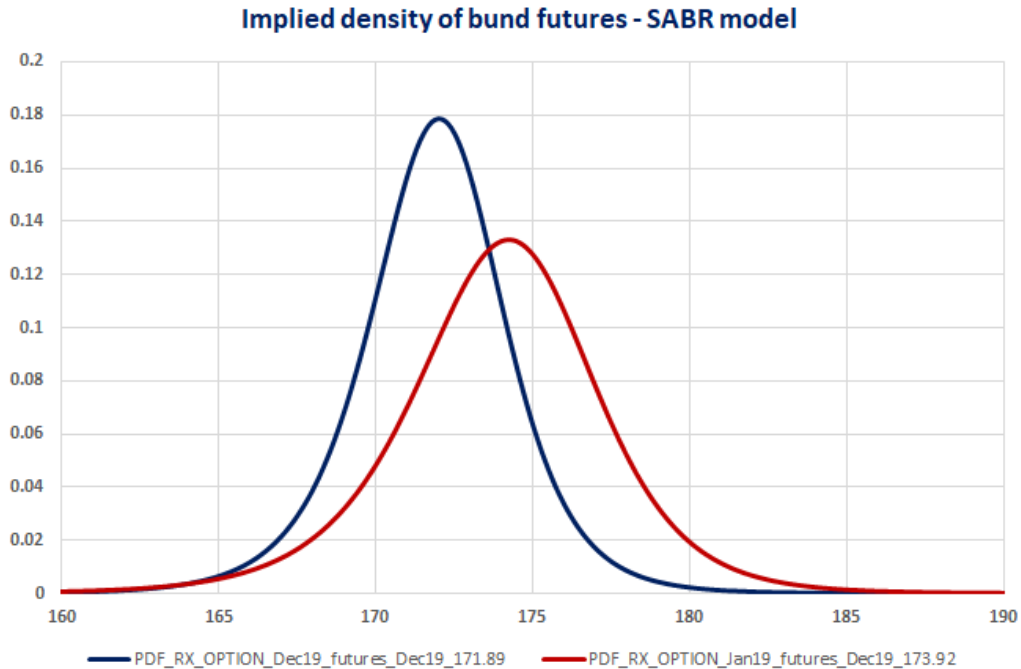


Figure 10: Implied density from two option contracts on Bund futures as of 15 October 2019 (December 2019 option (RXZ9) with December 2019 Bund futures underlying (RXZ9) and January 2019 option (RXF0) with March 2020 Bund futures underlying (RXH0)). The density is calculated by fitting SABR model to market implied volatilities. *Source: Bloomberg, authors' calculations.*

We illustrate how changes in the shapes of the distributions reflect changes in market expectations by comparing specific dates in recent history. During the course of 2019, the implied volatility of Bund futures was relatively subdued, oscillating around 4% as Bund yields gradually trended downwards. On 1 August 2019, the 30-day implied volatility index reached 3.7% annualised (or 41 basis points). The implied density of October 2019 option (RXV9) on Bund futures, with expiry after the September ECB Governing Council meeting, exhibited a narrow shape and a higher peak at the centre suggesting lower level of uncertainty by historical comparison. Yields fell further in August and reached all-time lows on expectations of further ECB easing and global trade concerns. On 19 August 2019, the implied volatility index increased to 5.8% p.a. (66 basis points) and the implied density from options on Bund futures with the same expiry (RXV9) confirmed the increase in perceived uncertainty regarding the upcoming ECB monetary policy decision, among other things. Its tails became heavier and peak at the centre smaller, despite shorter time to maturity. The change in the shape of the density may have also been amplified by the usual decline in liquidity in August. A lower liquidity in terms of lower amount of traded contracts and wider bid-ask spreads hints at an increased caution in interpreting the results precisely, in particular when the input data are scarce. Rather than clinging to exact probabilities, it may be more useful to interpret the results in terms of the change in the shape of the density, its asymmetry, and the change in the overall level of uncertainty.

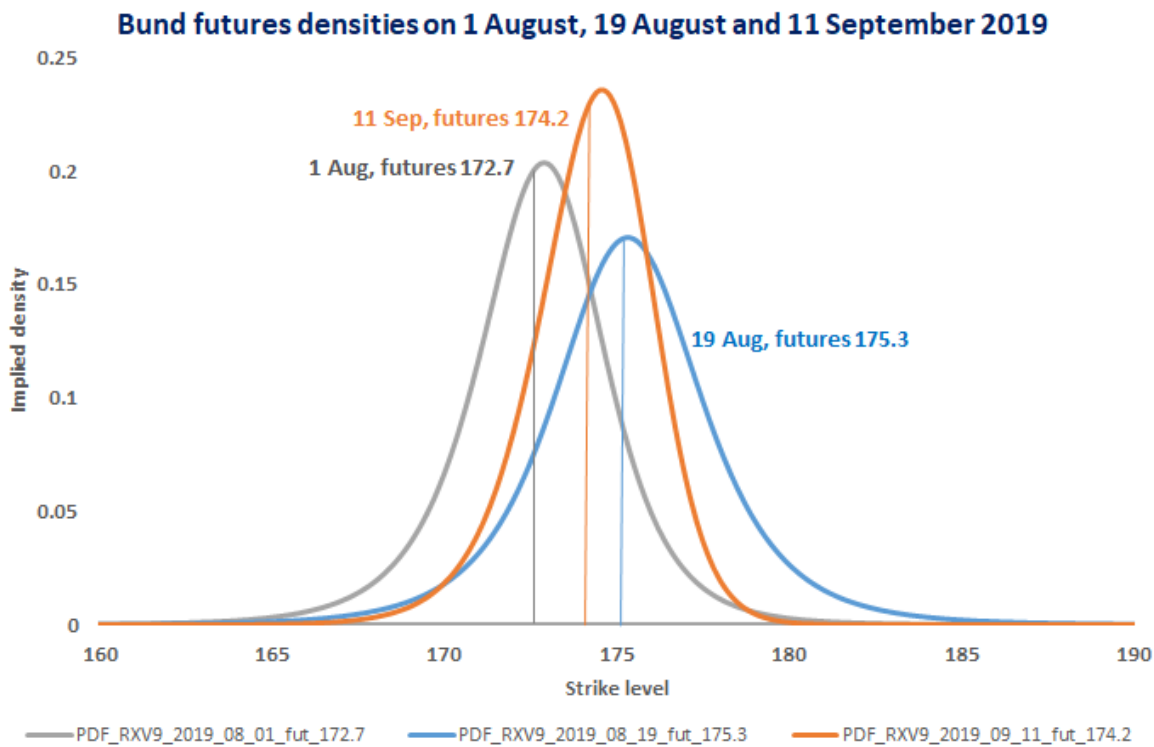


Figure 11: Implied Bund futures densities for selected dates ahead of the ECB GC September meeting. The level of uncertainty has shifted substantially higher during August before retreating somewhat. *Source: Bloomberg, authors' calculations.*

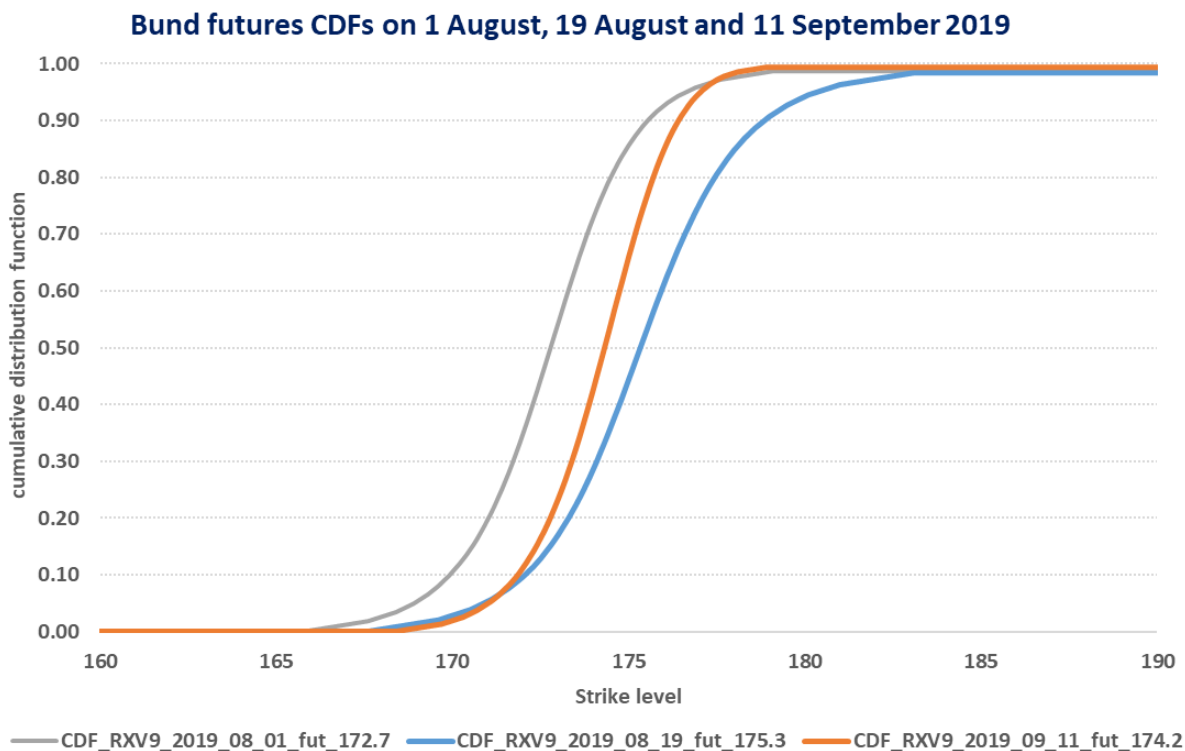


Figure 12: Implied cumulative distribution function of Bund futures for selected dates. *Source: Bloomberg, authors' calculations.*

Constant maturity distribution

Since options have defined expiration dates, it is useful to construct an interpolated constant maturity distribution for historical analysis, for example, with one-month maturity. We can construct a 30-day constant maturity distribution by interpolating the implied volatilities of two options with closest expiration dates below and above 30 days as in (4).

First, we fit the volatility smile with SABR model for the two options, generating equally spaced volatility-strike pairs with enough granularity. Next, we interpolate between the two smoothed volatilities using time-weighted linear interpolation as in (4)

$$\sigma_{30 \text{ day},i} = w\sigma_{1,i} + (1 - w)\sigma_{2,i} \text{ for } i = 1, \dots, n \quad (13)$$

where weight $w = \frac{T_2 - \frac{30}{365}}{T_2 - T_1}$, and T is option's time to expiration expressed as year fraction, and n is the number of interpolated volatilities.

Then we linearly interpolate between the two options' underlying Futures prices, F_1 and F_2

$$F = wF_1 + (1 - w)F_2 \quad (14)$$

and convert the interpolated volatilities $\sigma_{30 \text{ day}}$ back into option prices using interpolated futures price F and 30-day maturity. Finally, we can calculate the 30-day constant maturity PDF from (11).

Calculating quantiles

Tracking implied quantiles over time can be useful for monitoring changing probabilities of more or less extreme outcomes. We can calculate quantiles of the distribution from the cumulative distribution function - CDF, in (9) and the PDF in (11). The CDF can be recovered from the sum of its densities

$$CDF(K_N) = \sum_{i=1}^N f(K_i) \quad (15)$$

Since the quantile is the inverse CDF, we can recover strikes K_N for specific probabilities (quantile function returns the level of strike for a given option-implied probability; with this probability, the underlying bond futures will be at or below a certain strike).

4.1. Empirical analysis of option-implied distributions

We can now examine option-implied distributions and uncertainty surrounding bond futures in both a historical and a future context.

Analysing uncertainty over time

To analyse uncertainty over time, we construct a constant 30-day maturity chart with different quantiles of the option-implied distribution, which show more and less probable outcomes. The difference between the quantiles indicates the changing range of strikes that underlying futures will reach with given probabilities. This may be relevant from a risk management perspective since we can calculate the market pricing of a magnitude of the decline at a given probability.

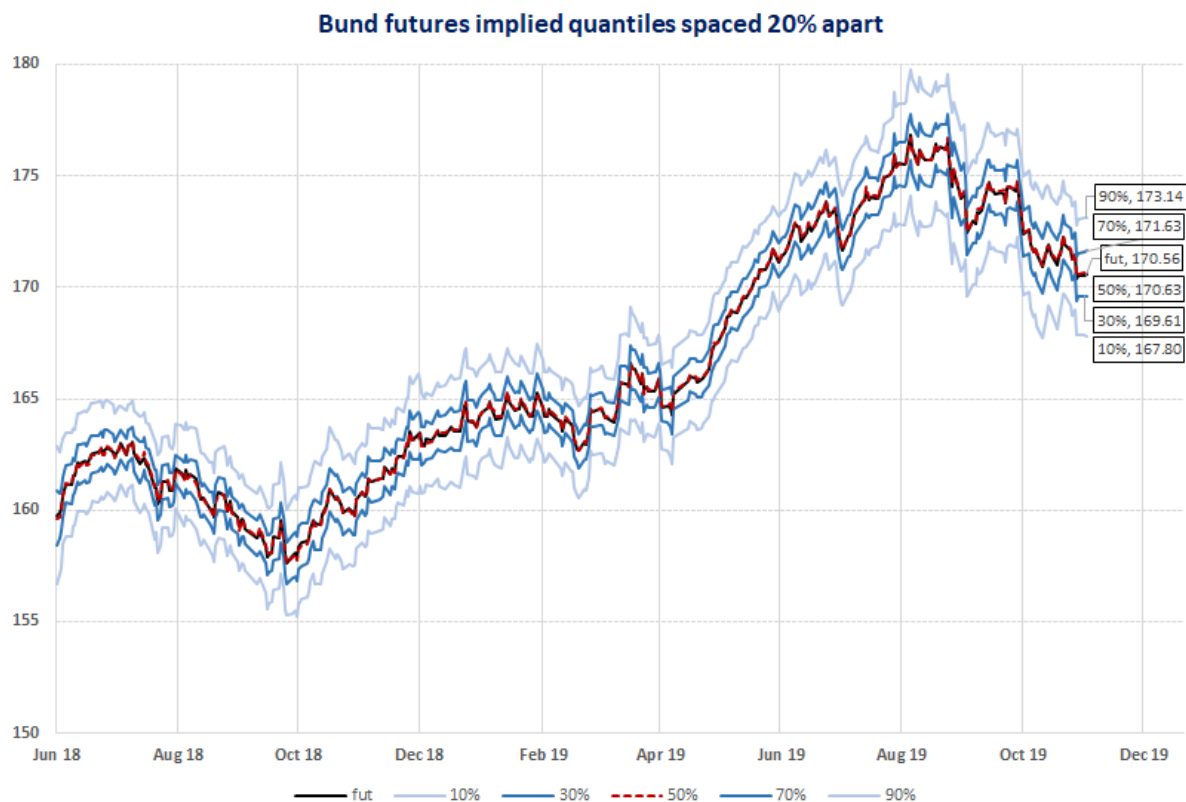


Figure 13: The history of constant 30-day maturity quantiles of implied Bund futures distribution. Different shades of blue separate the quantiles starting from 10%, ending at 90%, and spaced 20% apart (from less to more probable outcomes). The y-axis shows the strike at a given probability level; for example, 10% quantile is represented by the lightest blue line and shows the level of the strike that will be reached in 30 days with a 10% probability. *Source: Bloomberg, authors' calculations.*

From a portfolio management perspective, it may be useful to estimate option-implied probabilities that futures prices (or yields) rise or fall by a certain amount, or hit a specific level. We can retrieve such measures directly from the CDF in (12) by setting the strike K_N at, for example, 1% or 5% lower than futures price F . The graph below shows the implied probability that Bund futures drop by 1% or 5%, in the next 30 days.

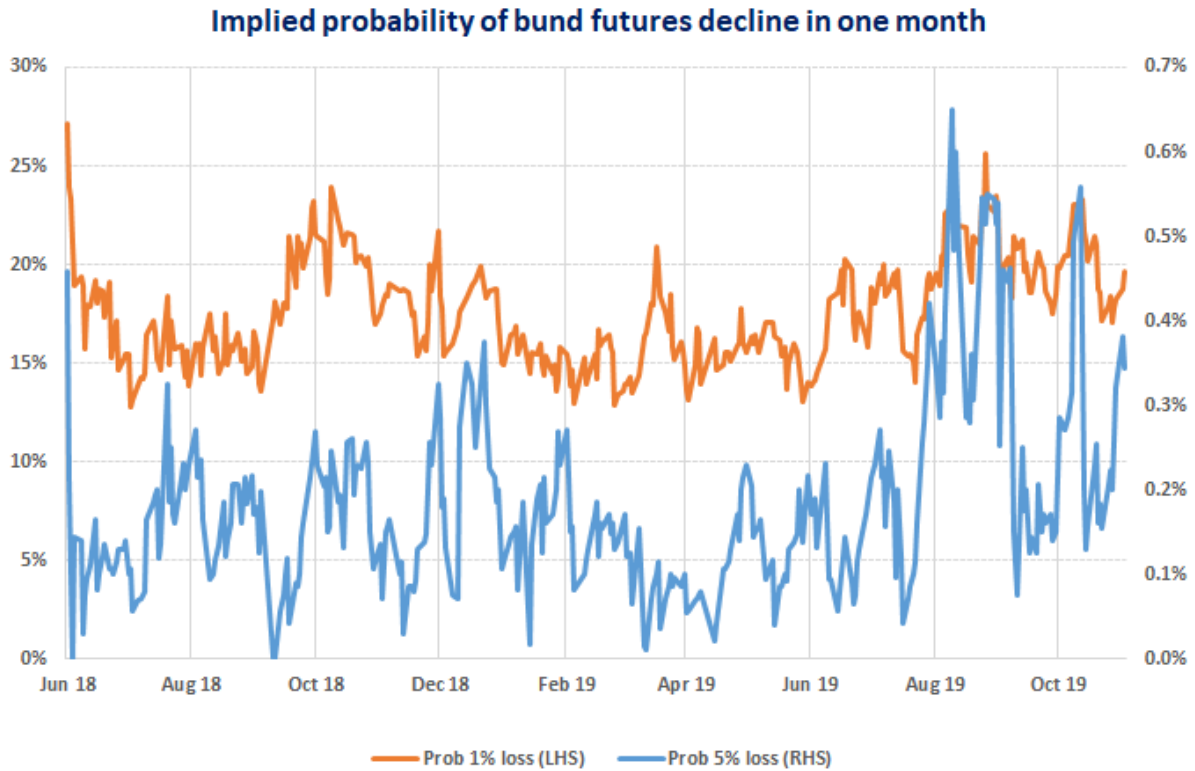


Figure 14: Implied probability of Bund futures falling by 1% (LHS) or 5% (RHS) in the next 30 days.
Source: Bloomberg, authors' calculations.

The probability that Bund futures fall by 1% in one month has been oscillating between 13% and 25% and increases with rising implied volatility. It is often more intuitive to express such loss probability in terms of change in yield in the underlying bond rather than change in the price of the futures contract. A 1% change in Bund futures price corresponds to about 11 basis point change in the yield of the underlying CTD Bund, on average. We can use price-yield sensitivity relationship from (5) to estimate implied probability of gain or loss expressed in basis points change

$$\Delta y = -\Delta F \times \frac{100}{DV01_F}, \quad (16)$$

where Δy is the basis points change in yield, ΔF is futures prices absolute change, and $DV01_F$ is futures DV01. Implied uncertainty expressed as probability of change in yields over time horizon may be more practical to use.

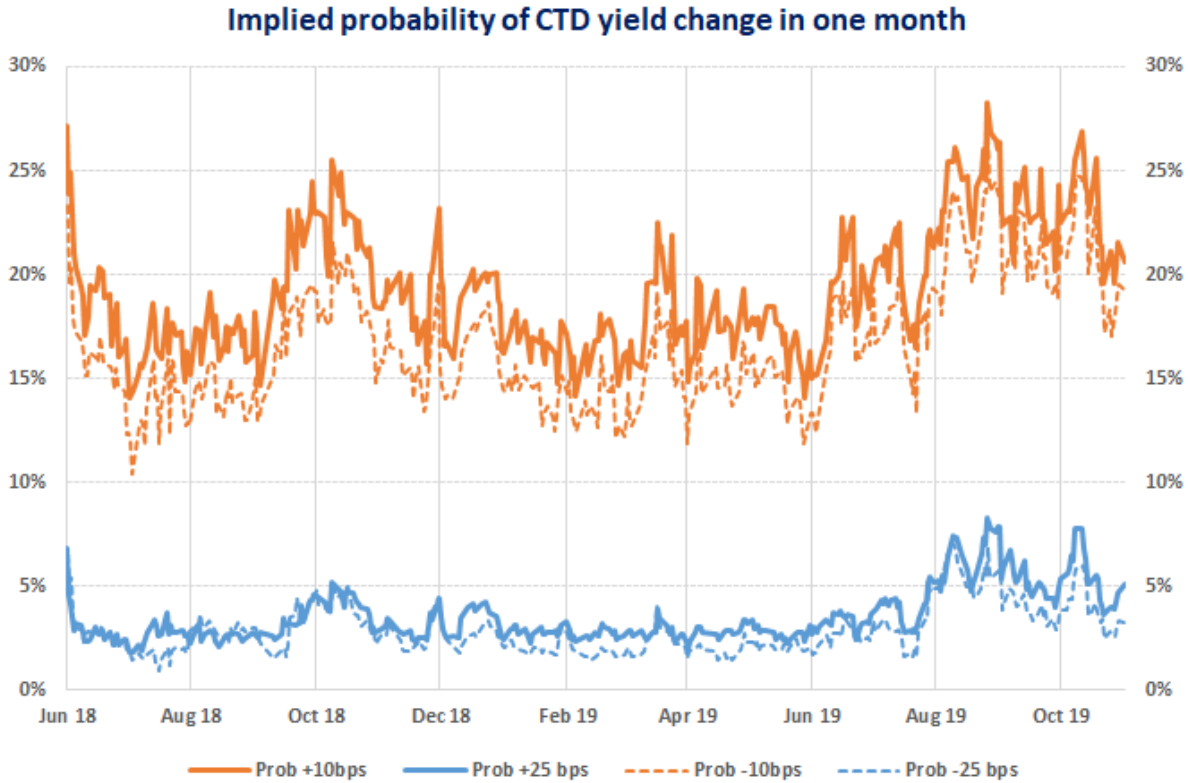


Figure 15: Implied probability of 10 (orange) and 25 (blue) basis points change in yields in CTD bonds. Solid (dashed) line shows the probability of yield increase (decrease). *Source: Bloomberg, authors' calculations.*

We can observe that, in the case of Bund futures, the implied distribution is priced towards higher yields (lower futures prices); an increase in yields is on most occasions priced as more likely than the decrease in yields. Probabilities have increased since August 2019, consistent with an increase in implied volatility.

Option-implied skewness and kurtosis

Similar to risk reversal strategy, we can calculate skewness to measure the asymmetry of a distribution and assess the balance of risks between expected large downward and upward moves in bond futures. For example, a positive skewness would indicate higher option-implied probability of a large downward move in yields, than a large upward move.

We calculate the skewness from the mean, the standard deviation, and the third moment of the implied distribution

$$mean = \mu = \sum_{i=1}^n f(K_i)K_i \tag{17}$$

$$variance = \sum_{i=1}^n f(K_i)(K_i - \mu)^2 \tag{18}$$

$$standard\ deviation = sd = \sqrt{variance} = \sqrt{\sum_{i=1}^n f(K_i)(K_i - \mu)^2} \tag{19}$$

$$skewness = \frac{\sum_{i=1}^n f(K_i)(K_i - \mu)^3}{sd^3} \quad (20)$$

Another measure of the shape of the distribution and the uncertainty in the tail that complements high and low quantiles is kurtosis, calculated from the fourth moment and the standard deviation

$$kurtosis = \frac{\sum_{i=1}^n f(K_i)(K_i - \mu)^4}{sd^4} \quad (20)$$

A kurtosis measure reveals if the variance of the distribution comes from the tails or the central part. These additional measures provide useful insights into the asymmetry and tails of the distribution over time.

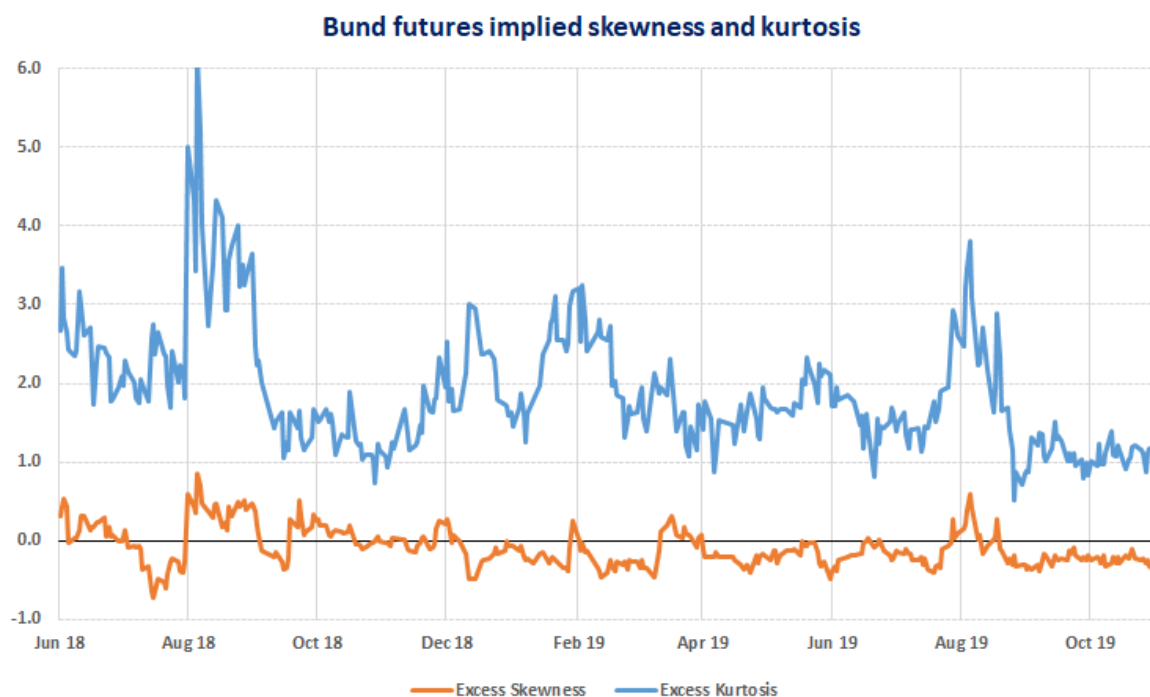


Figure 16: Option-implied skewness (orange) and kurtosis (blue) of Bund futures calculated from constant 30-day maturity distribution. Both are expressed in excess of skewness and kurtosis of lognormal distribution. *Source: Bloomberg, authors' calculations.*

The skewness of Bund futures has been mostly negative, indicating that the probability of a large sell-off (left tail of the distribution) is higher than the probability of a rally (right tail). Jumps in kurtosis indicate large expected price changes in the underlying Bund futures. A change in skewness has been associated with an increase in kurtosis, for example, in August 2019, the skewness turned positive, kurtosis and implied volatility increased, indicating a higher uncertainty and shift of tail probabilities from left to right.

We show skewness and kurtosis of the option-implied probability distribution for Schatz, Bobl, OAT and BTP futures, using constant maturity distribution of 30 days, in the Annex.

Analysing future uncertainty

A practical way to display future uncertainty is to create a fan chart with different option maturities on the x-axis and different quantiles on the y-axis, separated with colour shading, which indicates different levels of uncertainty. Fan charts help visualise probabilities associated with future outcomes, and the balance of risks to the futures price.



Figure 17: Historical Bund futures prices and their forecasted option-implied quantiles for August, September and October 2019 option maturities as of 8 August 2019 (the downward shift in expectations is caused by the roll in the underlying futures contract, from September to December contract). *Source: Bloomberg, authors' calculations.*

From the fan chart, we can observe that implied uncertainty increases with time to maturity and the balance of risks is slightly skewed to the downside (market pricing shows lower Bund futures prices are more likely than higher prices).

An option-implied distribution of futures prices at a particular date in the future, combined with a thorough survey across a spectrum of market participants can offer additional insights into the balance of risks at a future date and how materially the risks are being perceived. An option implied PDF can be, for example, combined with a histogram constructed from market participants' expectations. The shape of the histogram and the implied distribution reveals to what extent the market is sensitive to a particular event, such as a central bank policy surprise. This information cannot be extracted from futures or forward markets only.

One word of caution regarding surveys is that in many surveys, market participants are asked to express their expectations, i.e. their average forecast, and they do not forecast the distribution of

expected outcomes. A histogram constructed from a survey of such expectations then represents a distribution of these averages, and may be closer to normal distribution, i.e., such a histogram is likely to be more concentrated around the mean and exhibit lighter tails, and a lower standard deviation.

Conclusion

We expanded a family of volatility indices with new price and basis point volatility indices from options on European government bond futures and discussed their empirical properties. These indices reflect market pricing of subsequently realised volatility of the underlying futures. Volatility indices often correctly indicate the direction of subsequently realised volatility. We also compared correlations among volatility indices and their underlying bond futures and discussed the historical relationship between Bund and US Treasury implied volatility indices. After adjusting for the DV01, Bund implied volatility is on average lower than Treasury implied volatility. Such volatility indices could be used to measure market-implied uncertainty in European government bonds, and potential new instruments tracking them could offer a number of trading, hedging, and portfolio management opportunities.

We then showed how to accompany volatility indices with other useful information extracted from options prices, such as probability distribution of underlying futures, or its moments. These offer investors additional insights into the uncertainty around forward prices. We showed how probability distributions can be recovered from option prices and discussed their practical applications from risk and portfolio management perspectives. These included tracking quantiles over time, calculating moments of the distribution, probabilities of future outcomes, and displaying future uncertainty through fan charts. Option-implied uncertainty measures help market participants to form a comprehensive assessment of what is priced in the market.

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Annex - Chart pack of volatility indices and uncertainty measures from option-implied distributions of European government bond futures

1. Bobl futures

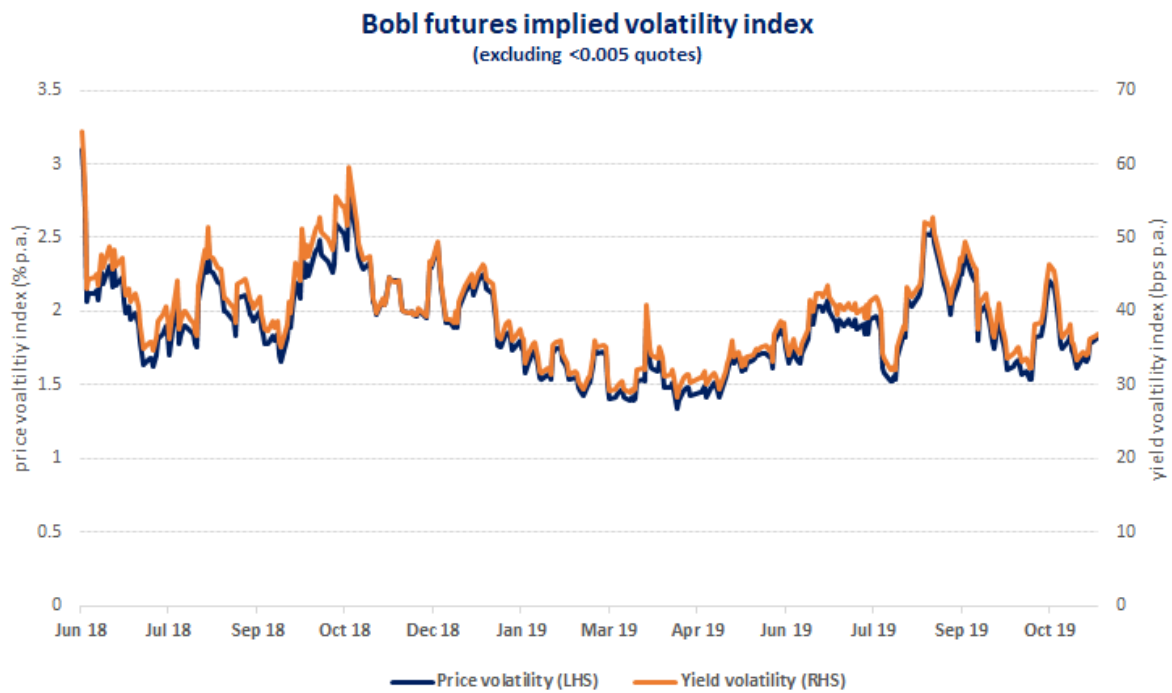


Figure 18: Implied volatility index of Bobl futures expressed in price (% p.a., LHS), and basis points (bps p.a., RHS). *Source: Bloomberg, authors' calculations.*

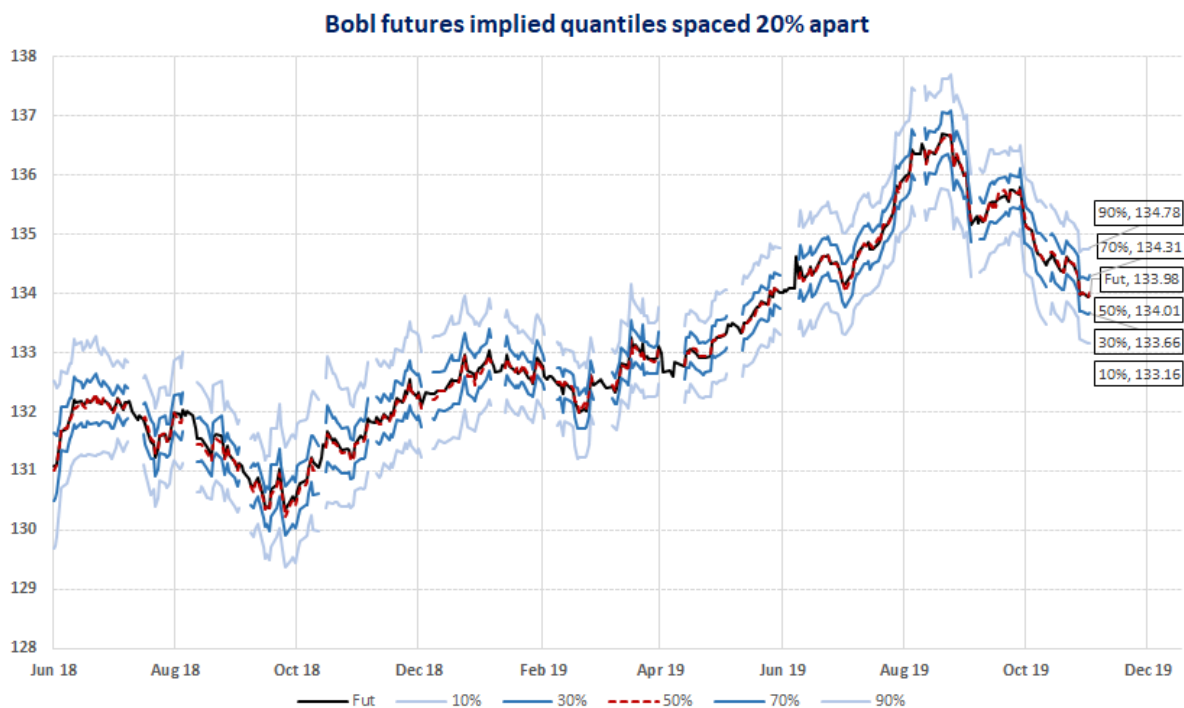


Figure 19: Implied quantiles of Bobl futures spaced 20% apart. *Source: Bloomberg, authors' calculations.*

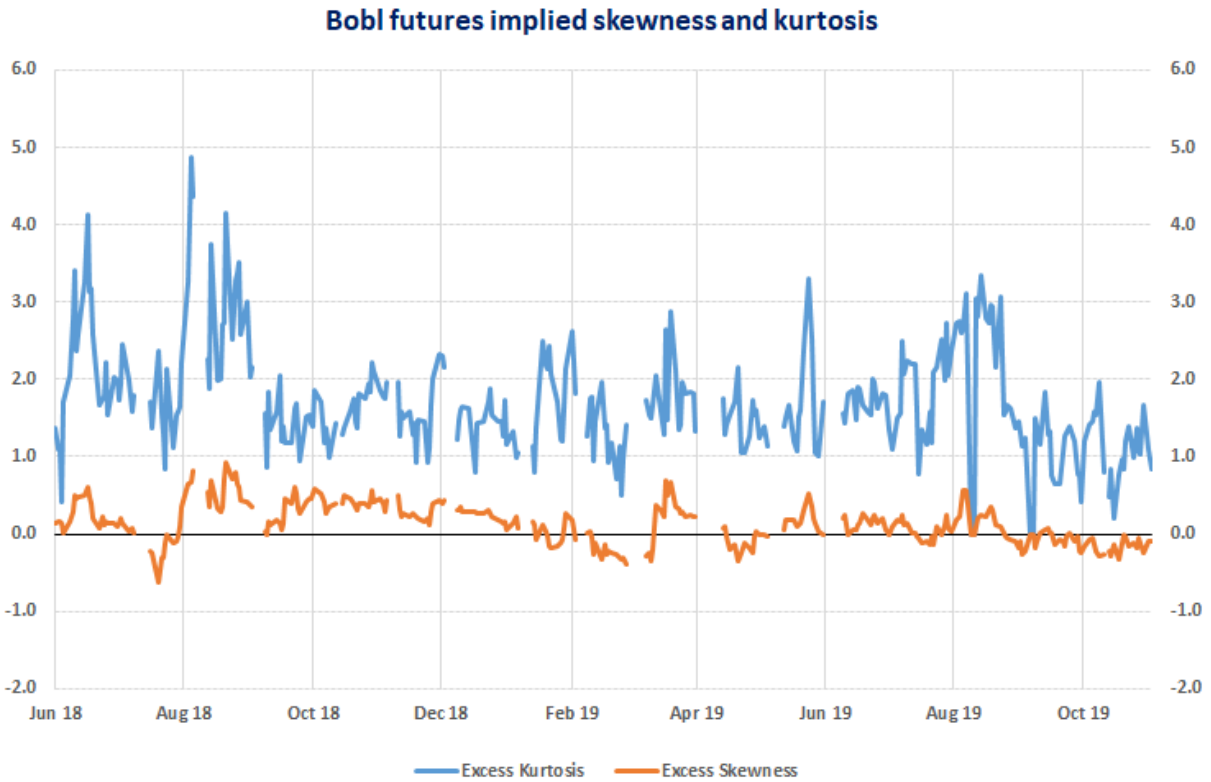


Figure 20: Implied skewness and kurtosis of Bobl futures, expressed in excess of skewness and kurtosis of lognormal distribution. Implied distribution of Bobl futures exhibits heavier tails than lognormal distribution. *Source: Bloomberg, authors' calculations.*

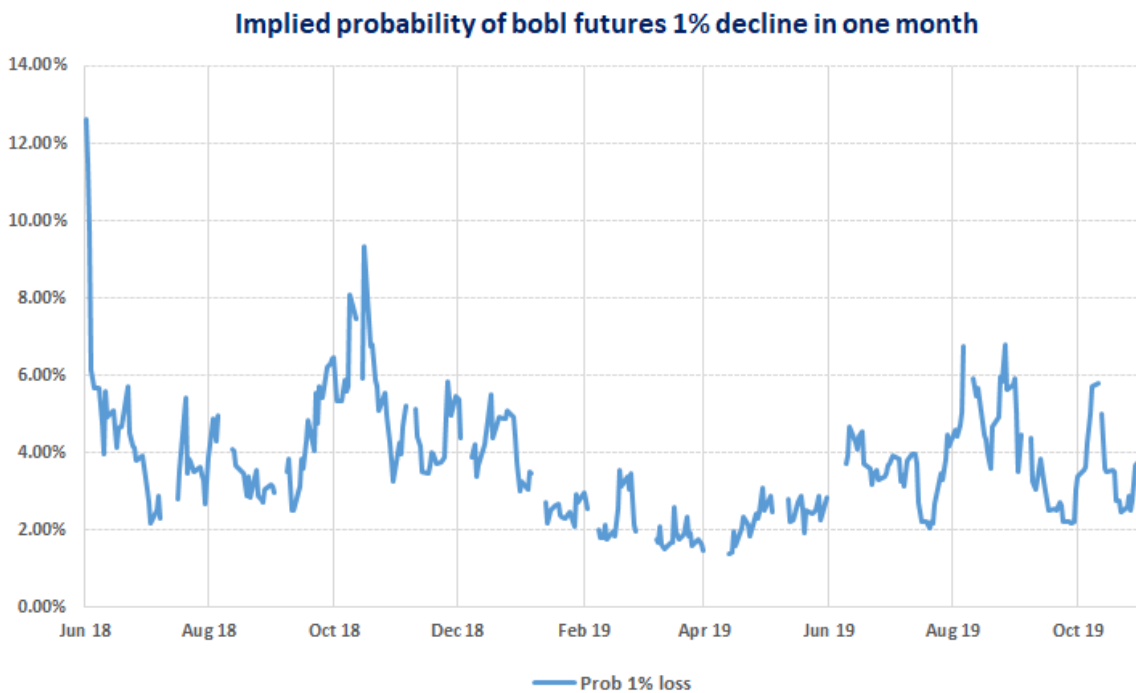


Figure 21: Implied probability of Bobl futures 1% price decline in one month. *Source: Bloomberg, authors' calculations.*

Implied probability of bobl CTD yield change in one month

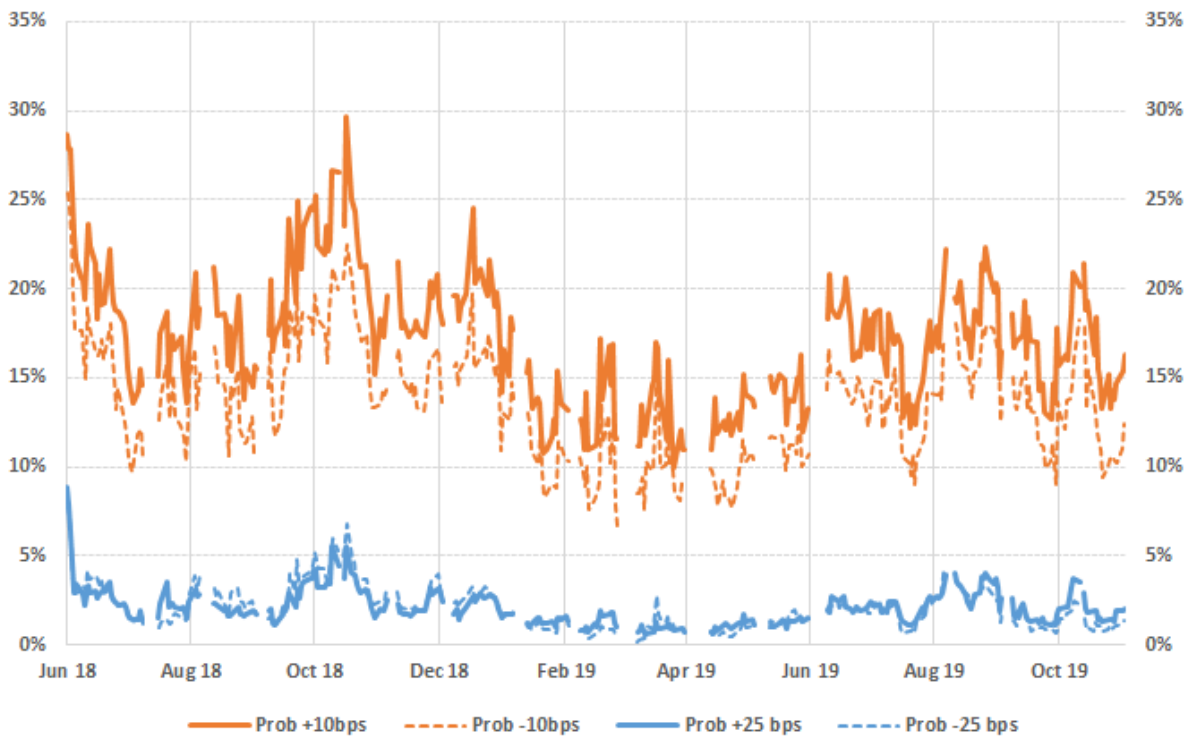


Figure 22: Implied probability of 10 (orange) and 25 (blue) basis points change in yields in CTD bonds of Bobl futures. Solid (dashed) line shows the probability of yield increase (decrease). *Source: Bloomberg, authors' calculations.*

2. Schatz futures

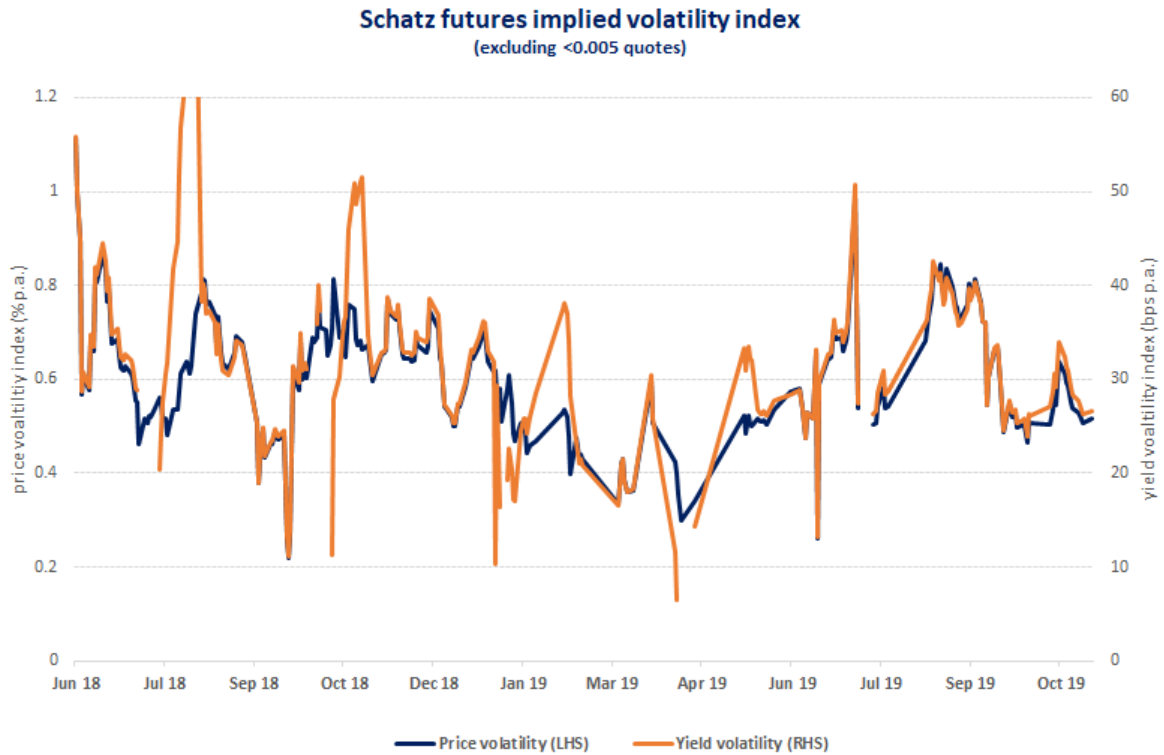


Figure 23: Implied volatility index of Schatz futures expressed in price (% p.a., LHS), and basis points (bps p.a., RHS). Low implied volatility for options close to expiration results in missing quotes for the near term options when the index cannot be produced. *Source: Bloomberg, authors' calculations.*

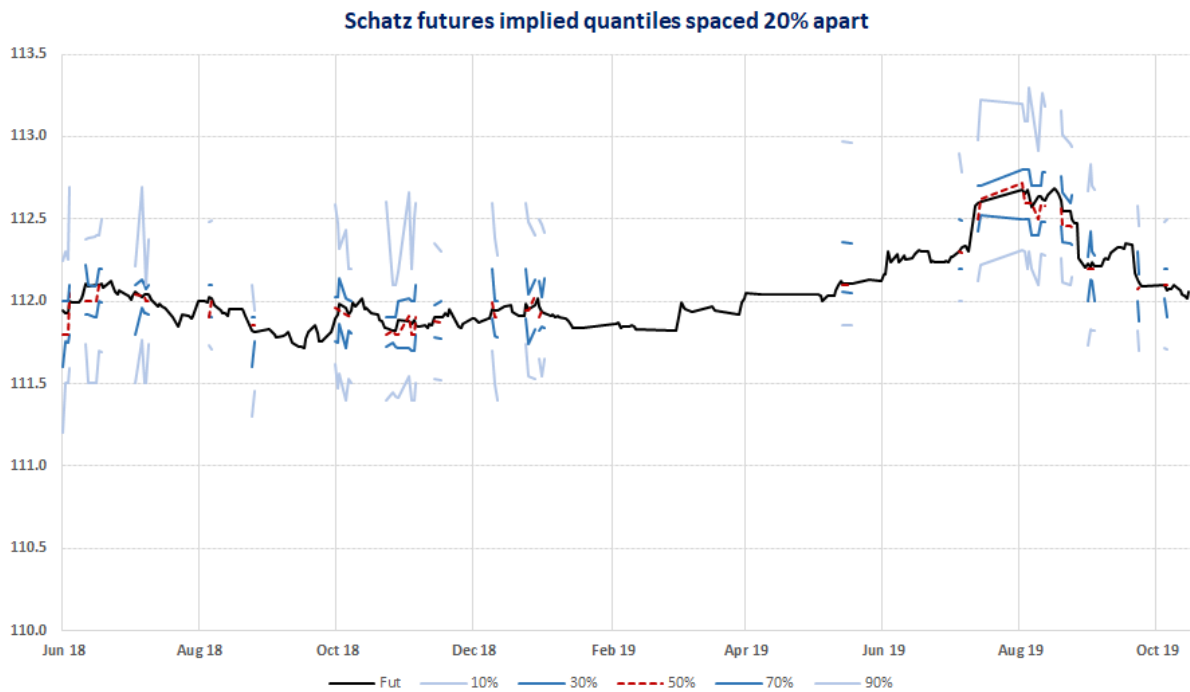


Figure 24: Implied quantiles from Schatz futures. Data scarcity in underlying options does not allow to calculate the implied distribution on most of the days. *Source: Bloomberg, authors' calculations.*

Schatz futures implied skewness and kurtosis

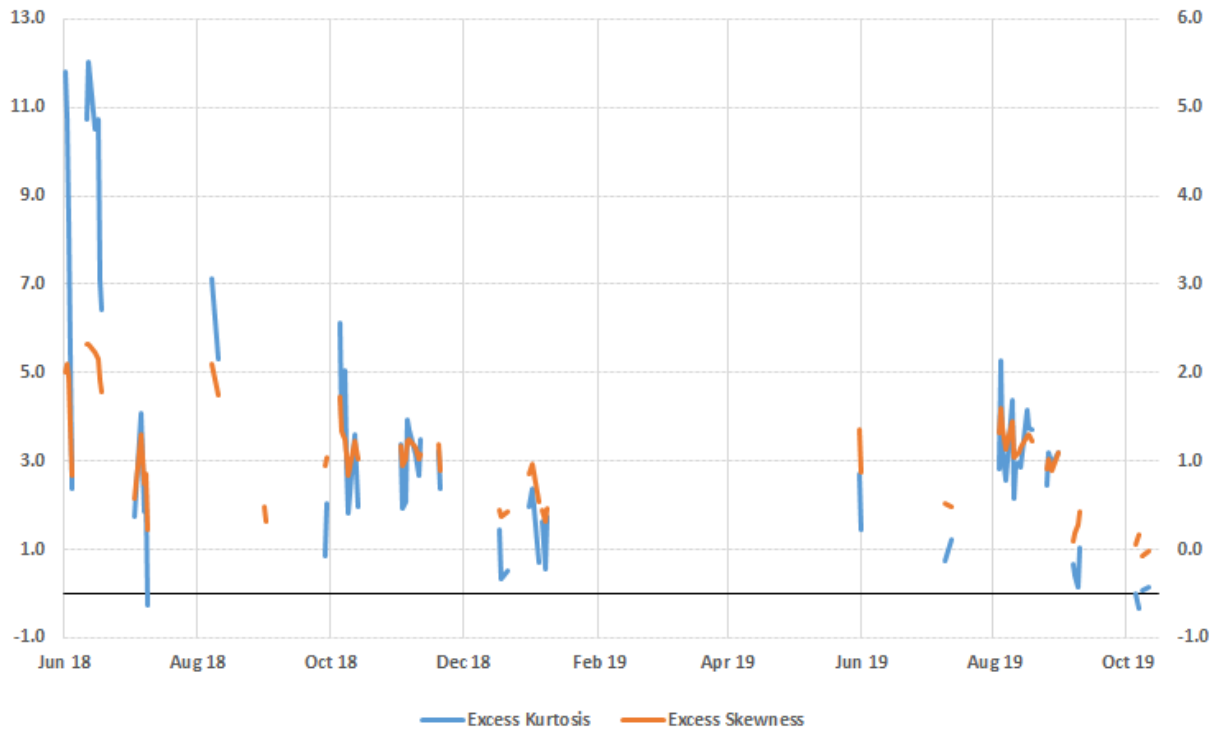


Figure 25: Implied skewness and kurtosis of Bobl futures, expressed in excess of skewness and kurtosis of lognormal distribution. The skewness has been mostly positive on the days with sufficient data to recover the implied distribution, suggesting flight-to-quality hedges. *Source: Bloomberg, authors' calculations.*

Implied probability of schatz CTD yield change in one month

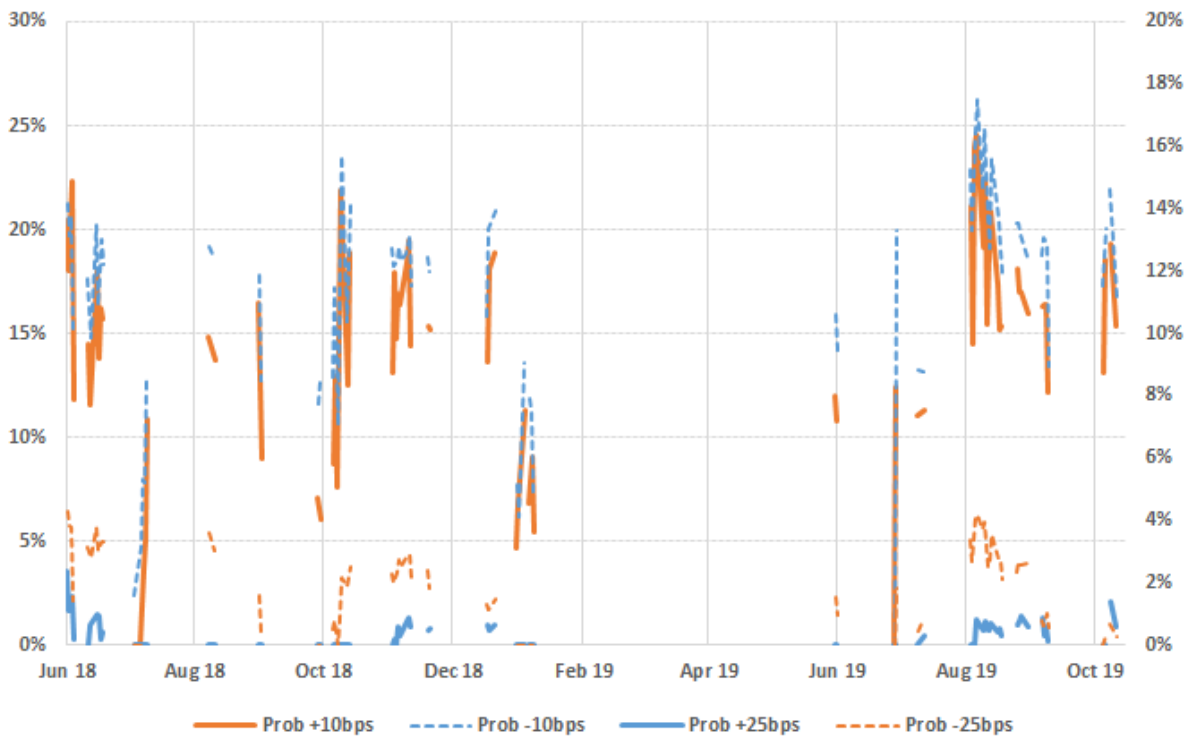


Figure 26: Implied probability of 10 (orange) and 25 (blue) basis points change in yields in CTD bonds of Schatz futures. Solid (dashed) line shows the probability of yield increase (decrease). Data scarcity makes this measure of little use in the case of Schatz. *Source: Bloomberg, authors' calculations.*

3. OAT futures

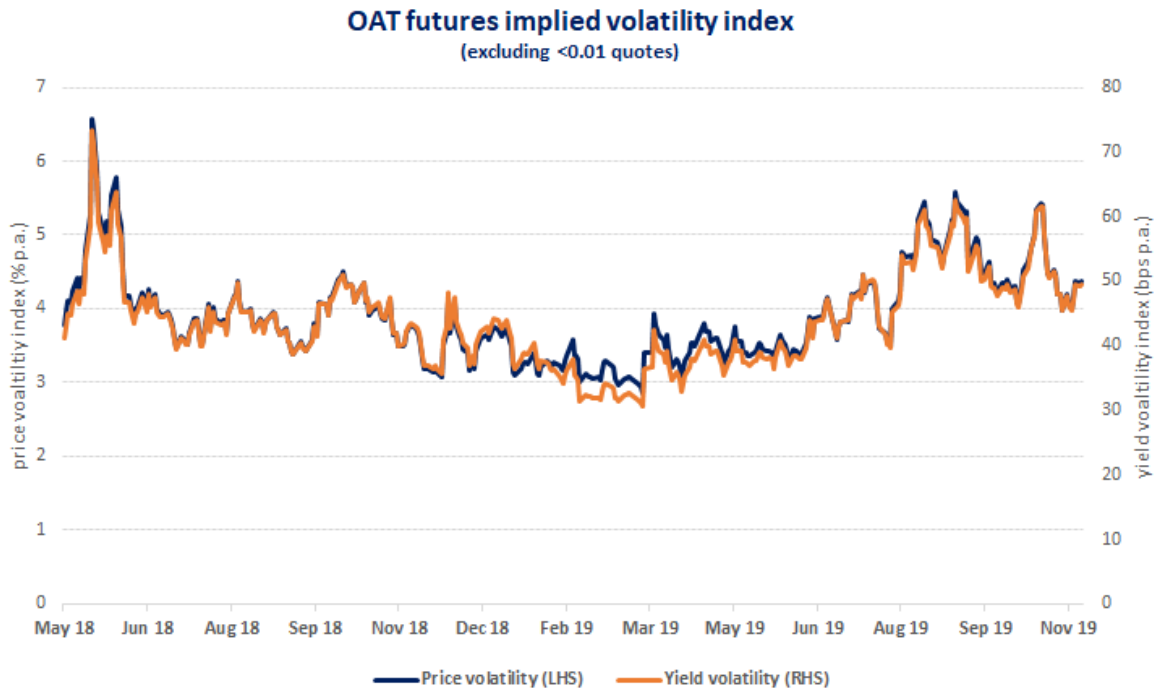


Figure 27: Implied volatility index of OAT futures expressed in price (% p.a., LHS), and basis points (bps p.a., RHS). *Source: Bloomberg, authors' calculations.*

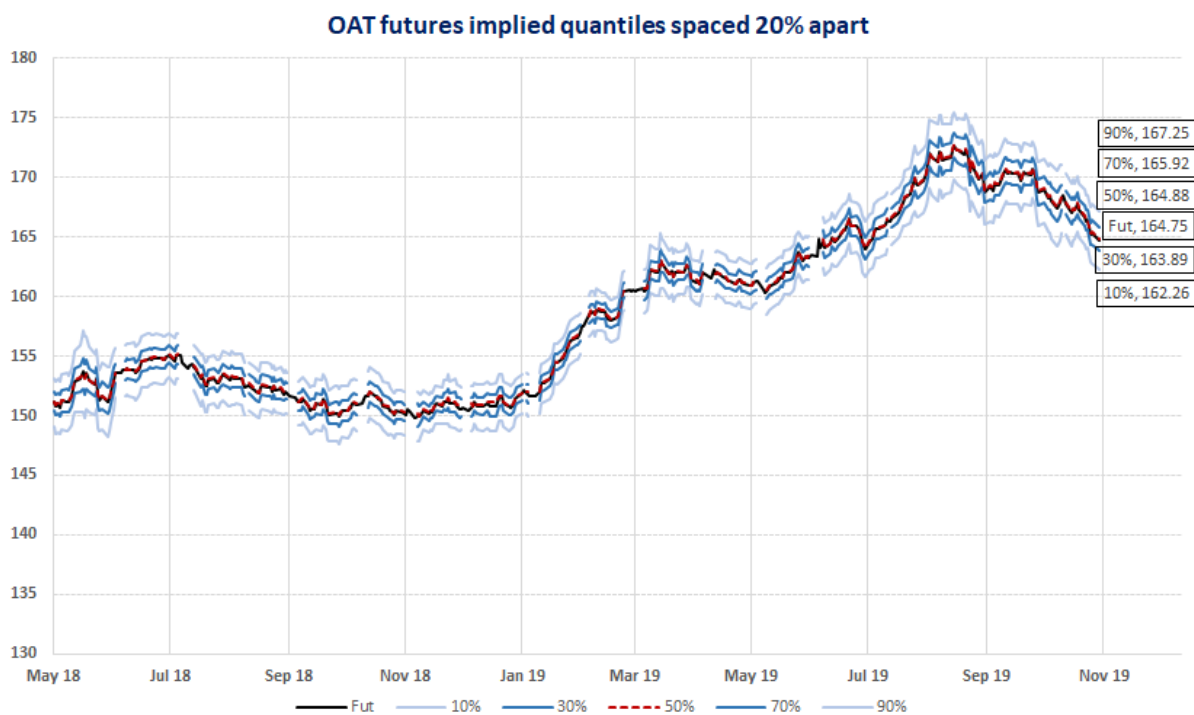


Figure 28: Implied quantiles of OAT futures spaced 20% apart. *Source: Bloomberg, authors' calculations.*

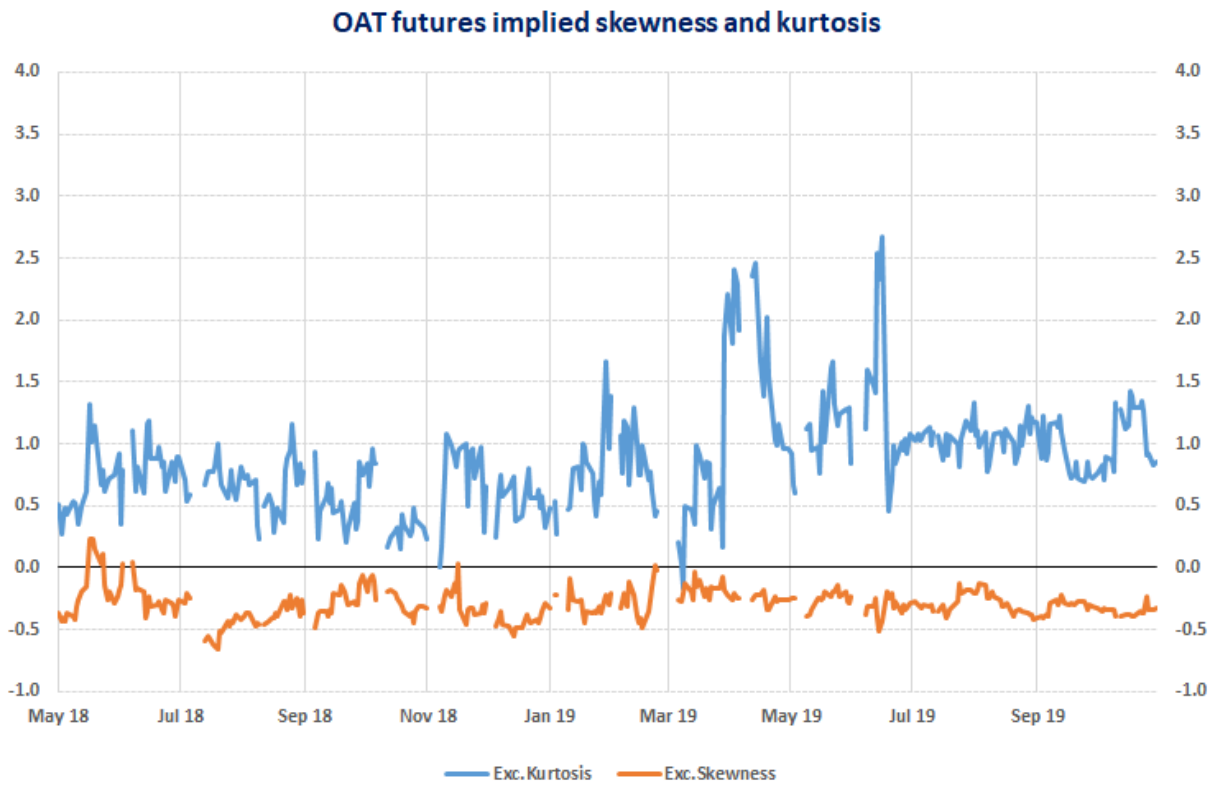


Figure 29: Implied skewness and kurtosis of OAT futures, expressed in excess of skewness and kurtosis of lognormal distribution. Negative skewness indicates higher probability of a large sell-off than a large rally. *Source: Bloomberg, authors' calculations.*

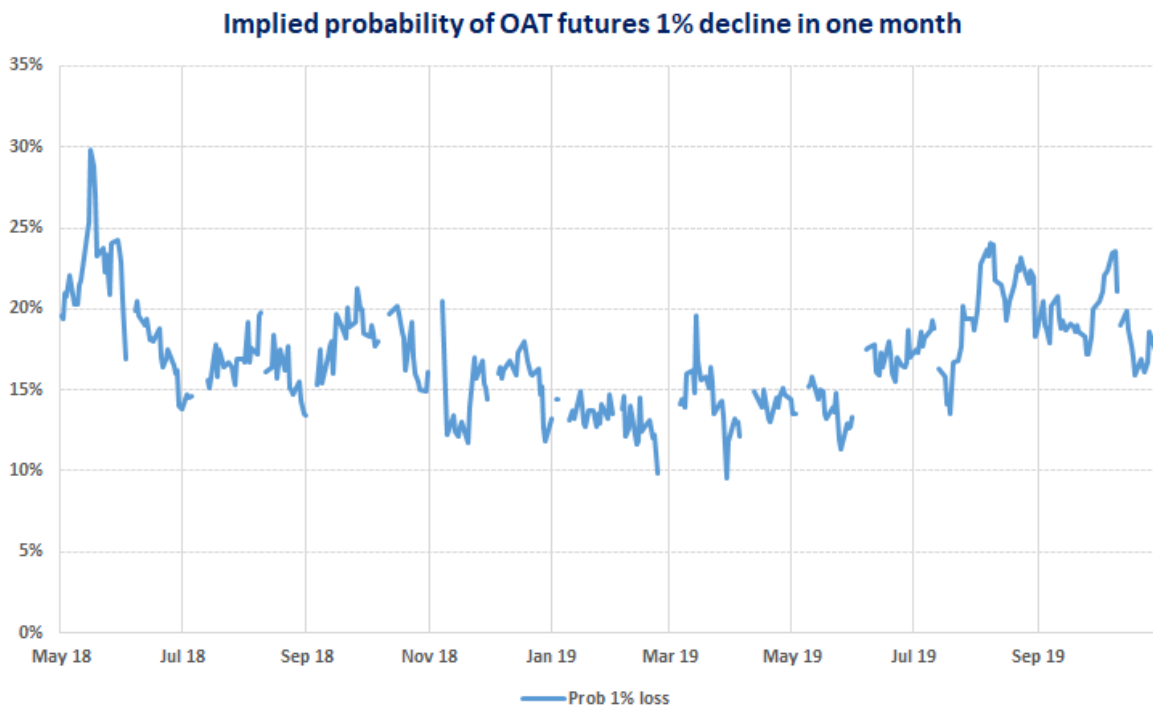


Figure 30: Implied probability of OAT futures 1% price decline in one month. *Source: Bloomberg, authors' calculations.*

Implied probability of OAT CTD yield change in one month

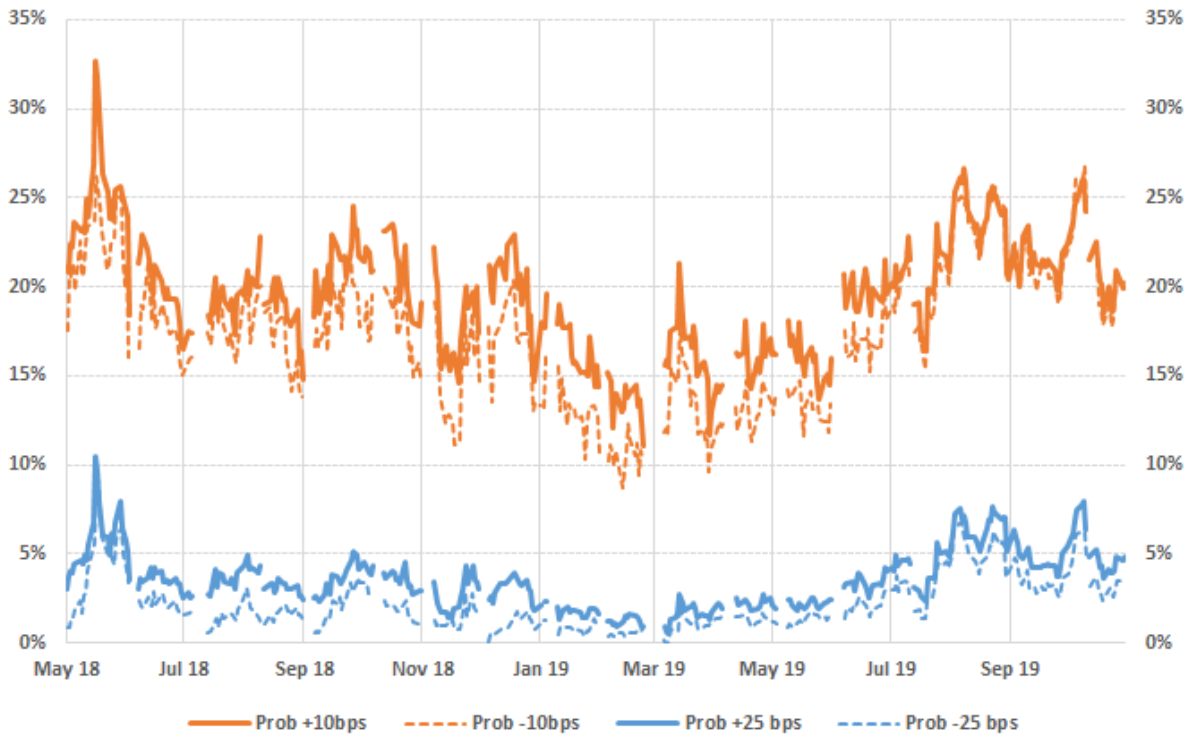


Figure 31: Implied probability of 10 (orange) and 25 (blue) basis points change in yields in CTD bonds of OAT futures. Solid (dashed) line shows the probability of yield increase (decrease). *Source: Bloomberg, authors' calculations.*

4. BTP futures

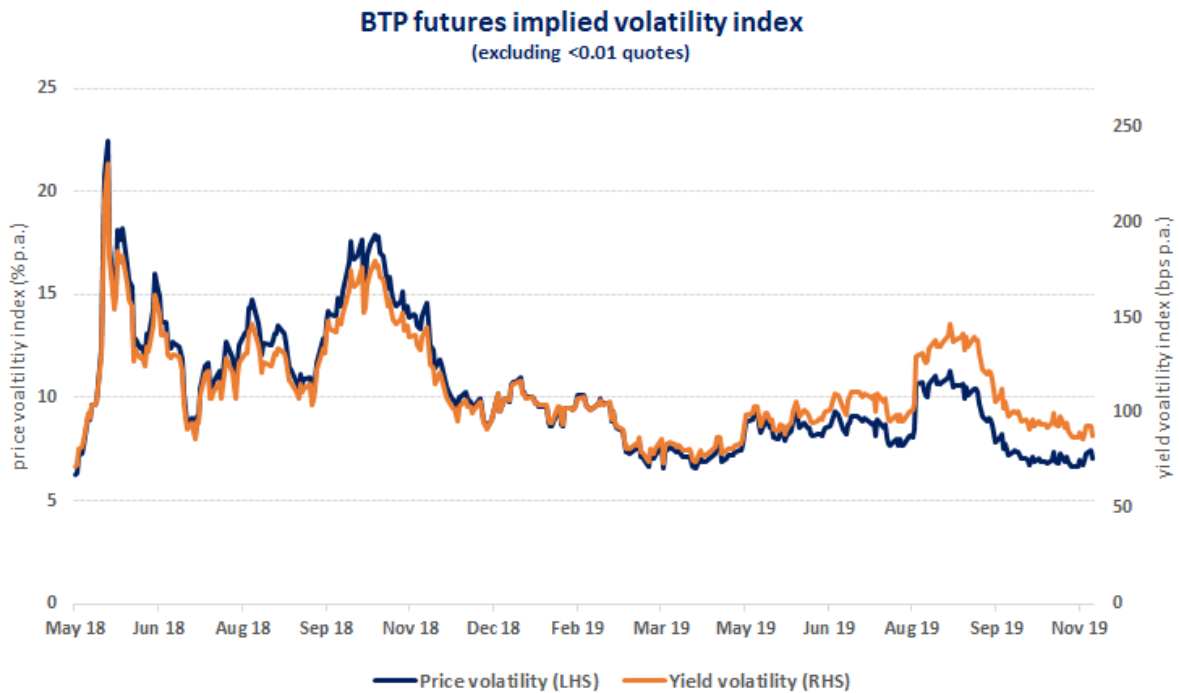


Figure 32: Implied volatility index of BTP futures expressed in price (% p.a., LHS), and basis points (bps p.a., RHS). *Source: Bloomberg, authors' calculations.*

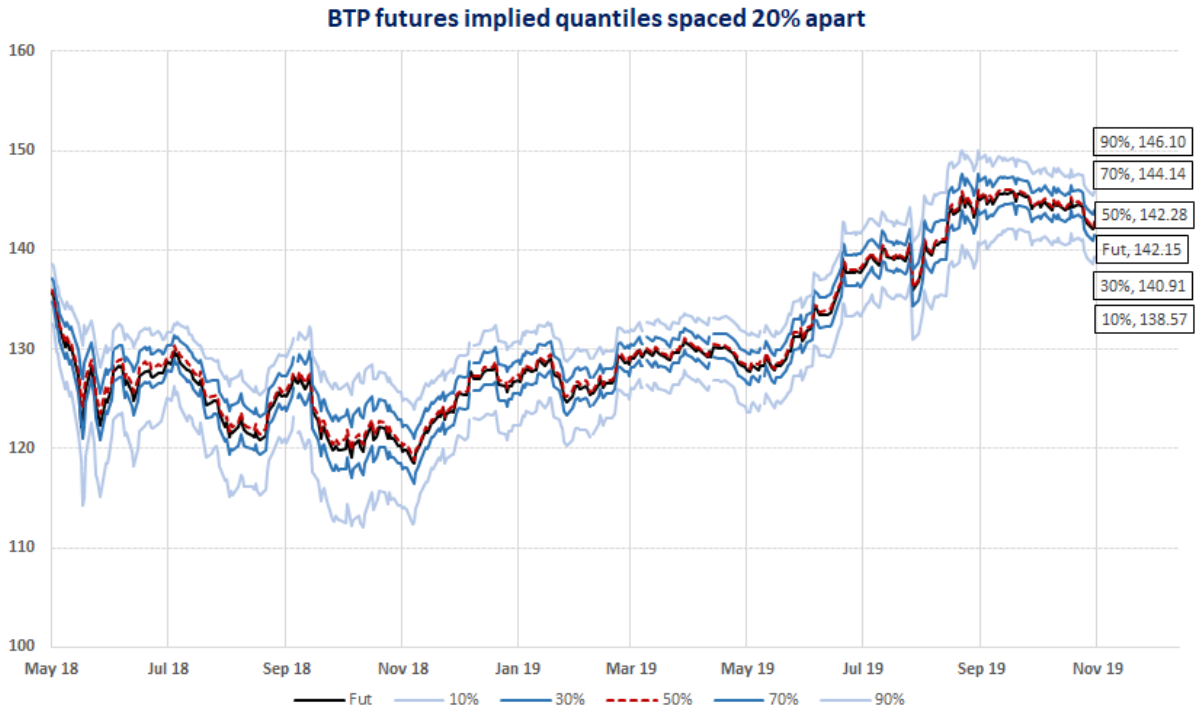


Figure 33: Implied quantiles of BTP futures spaced 20% apart. *Source: Bloomberg, authors' calculations.*

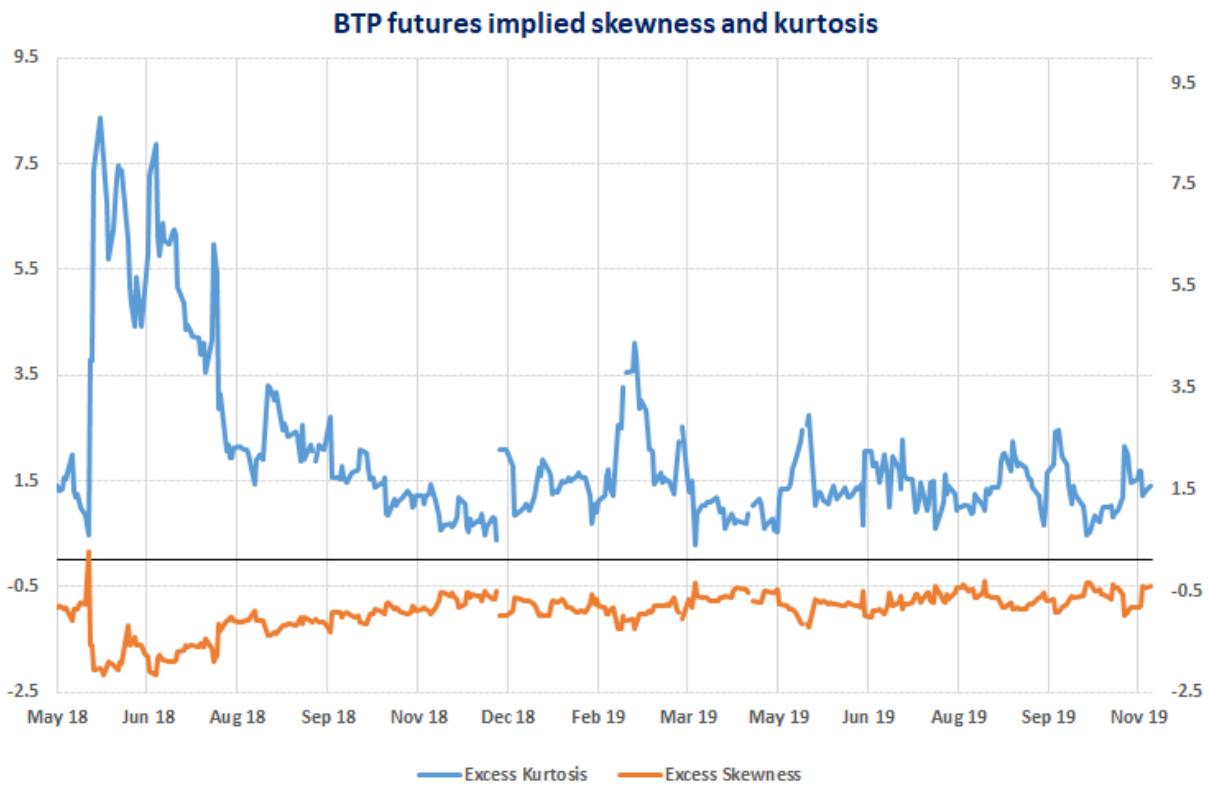


Figure 34: Implied skewness and kurtosis of BTP futures, expressed in excess of skewness and kurtosis of lognormal distribution. Tail risk probability in BTP futures is on average higher than OAT or Bund futures, and negative skewness show higher probability of a large sell-off (left tail) than a rally (right tail). *Source: Bloomberg, authors' calculations.*

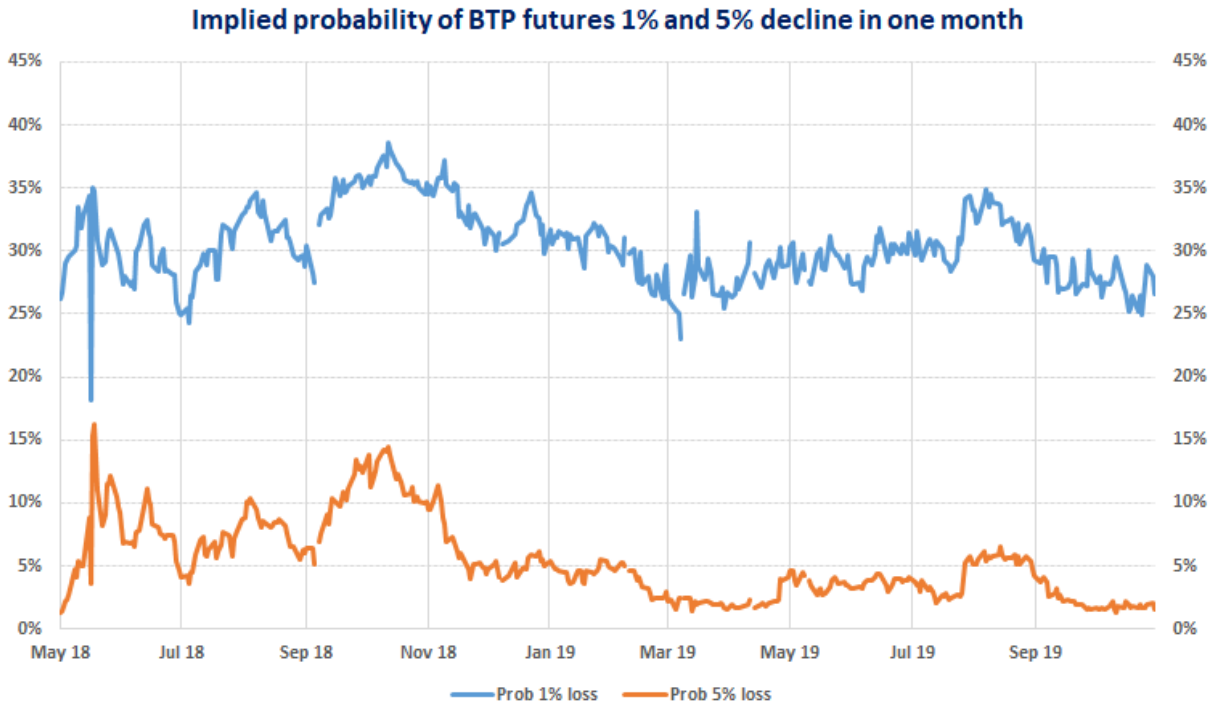


Figure 35: Implied probability of BTP futures 1% price decline in one month. *Source: Bloomberg, authors' calculations.*

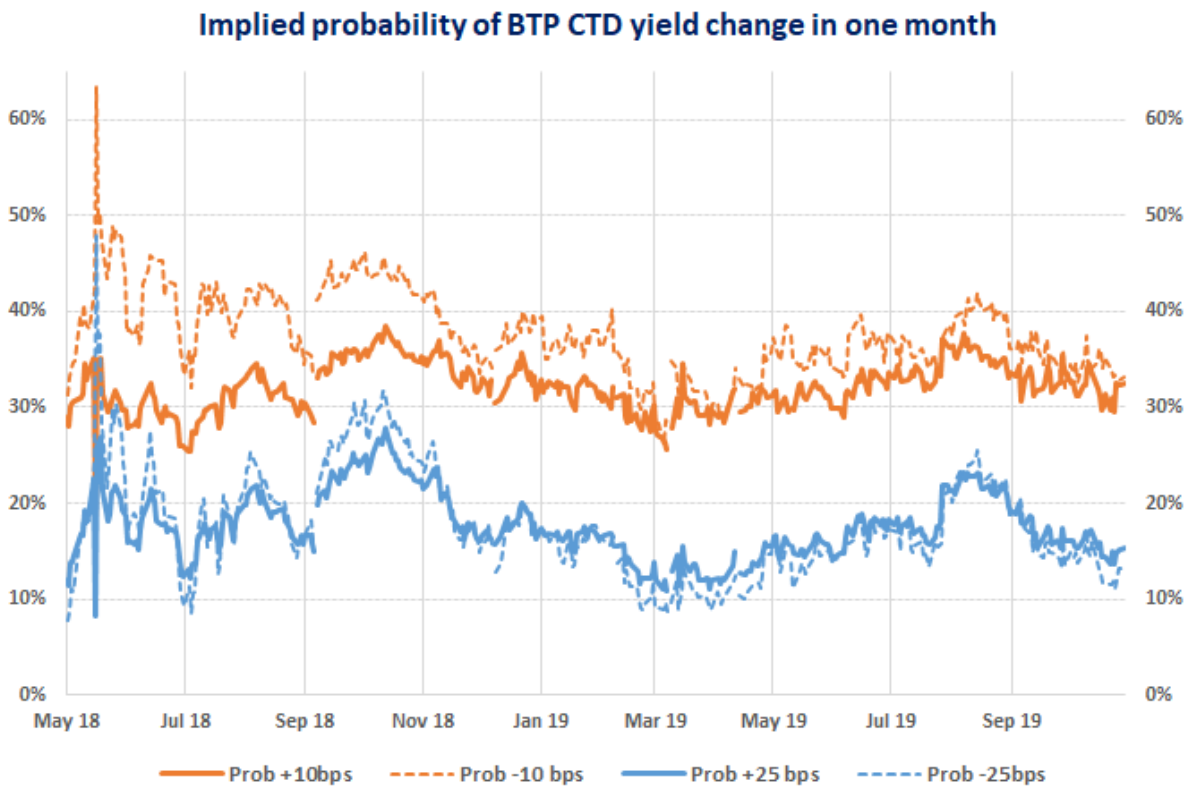


Figure 36: Implied probability of 10 (orange) and 25 (blue) basis points change in yields in CTD bonds of BTP futures. Solid (dashed) line shows the probability of yield increase (decrease). *Source: Bloomberg, authors' calculations.*

August 2018 BTP option-implied densities on selected dates

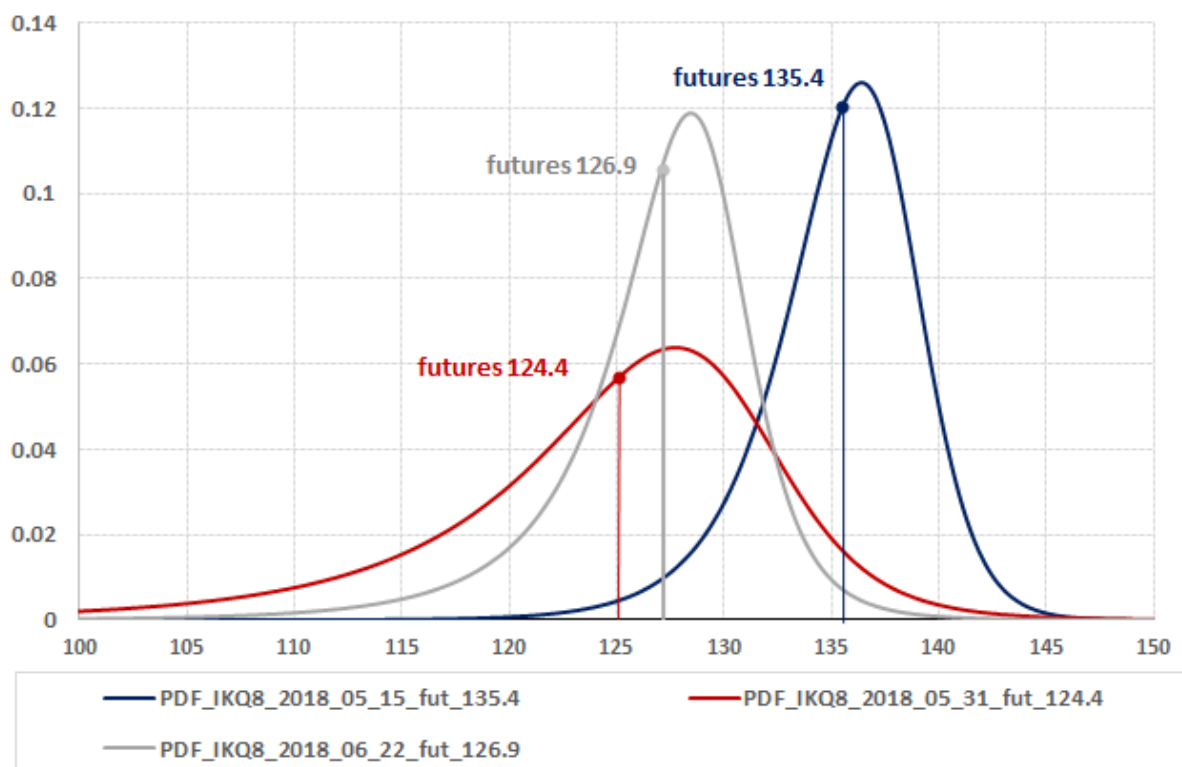


Figure 37: Implied density from August 2018 option contracts (IKQ8) on September 2018 BTP futures (IKU18) as of 15 May, 31 May and 22 June 2018. On 15 May 2018, 30-day implied volatility (IV) index stood at 6.3% p.a. (73 basis points). The density exhibited a narrow shape and higher peak at the centre; however, a negative skew indicated an early sign of downside expectations. On 31 May 2018, IV index jumped to 22.5% (231 bps), reaching historical extremes as uncertainty regarding Italian government formation culminated. The density tails became heavy and negative skew more pronounced. On 22 June 2018, IV index decreased to 12.9% (132 bps), and remained at elevated levels with more negative skew and heavier tails, relative to levels before the elections, despite shorter time to expiration. *Source: Bloomberg, authors' calculations.*

5. Bund futures - full history (2010-2019)

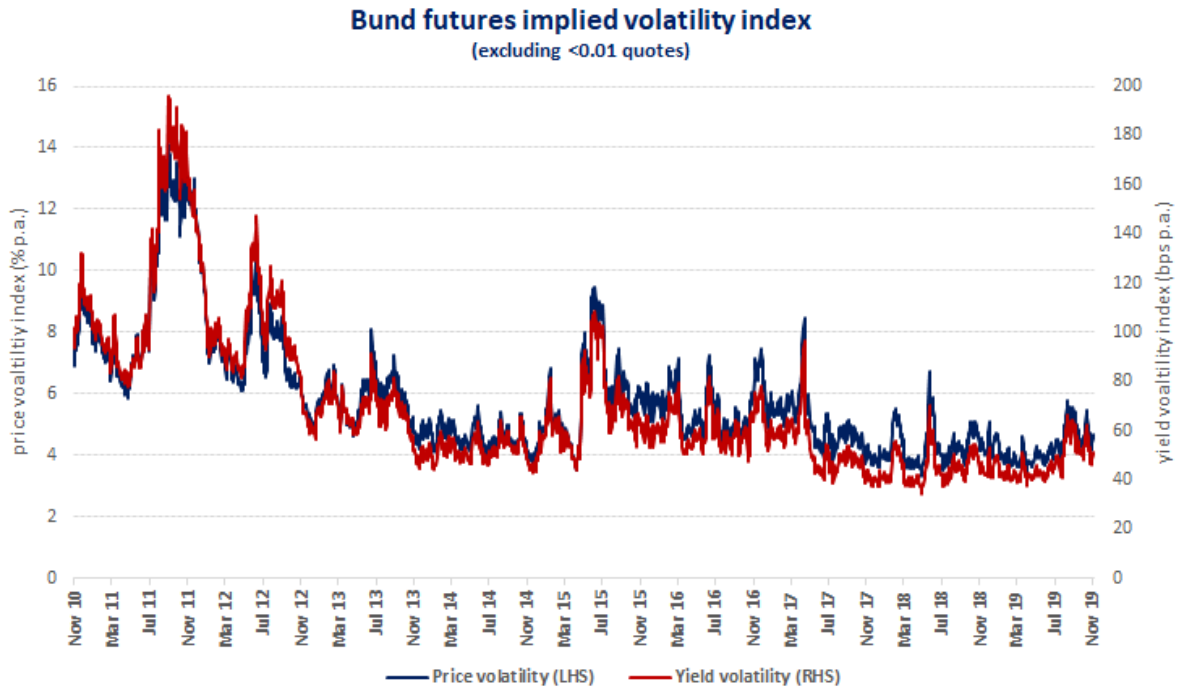


Figure 38: Implied volatility index of Bund futures expressed in price (% p.a., LHS), and basis points (bps p.a., RHS). *Source: Bloomberg, authors' calculations.*

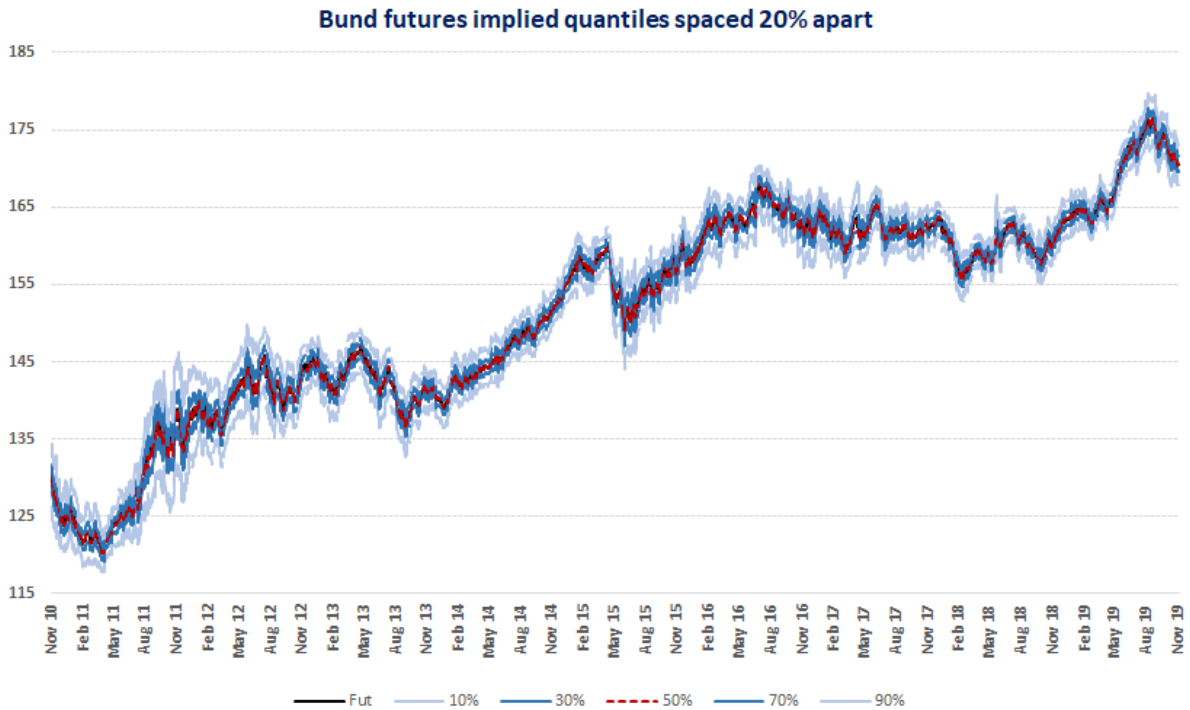


Figure 39: Implied quantiles of Bund futures spaced 20% apart. *Source: Bloomberg, authors' calculations.*

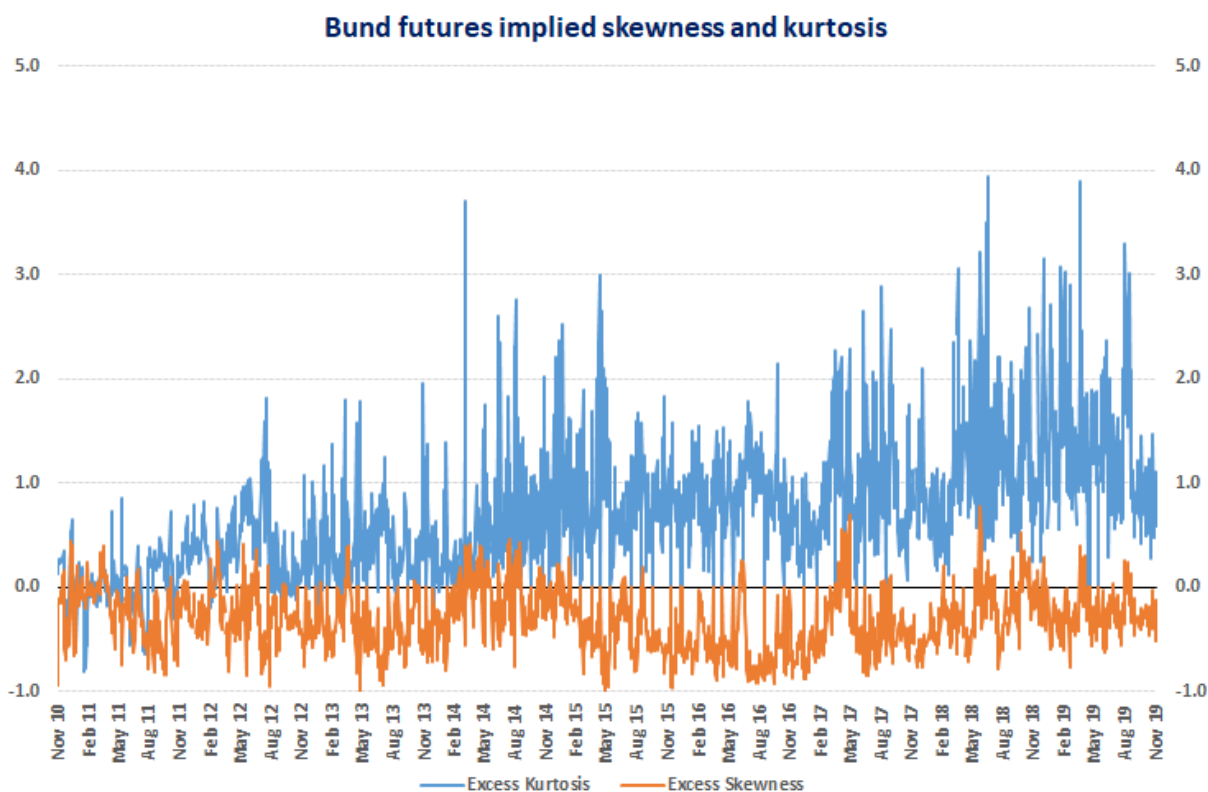


Figure 40: Implied skewness and kurtosis of Bund futures, expressed in excess of skewness and kurtosis of lognormal distribution. Average excess skewness over the full sample is -0.3, while average excess kurtosis is +0.7. *Source: Bloomberg, authors' calculations.*

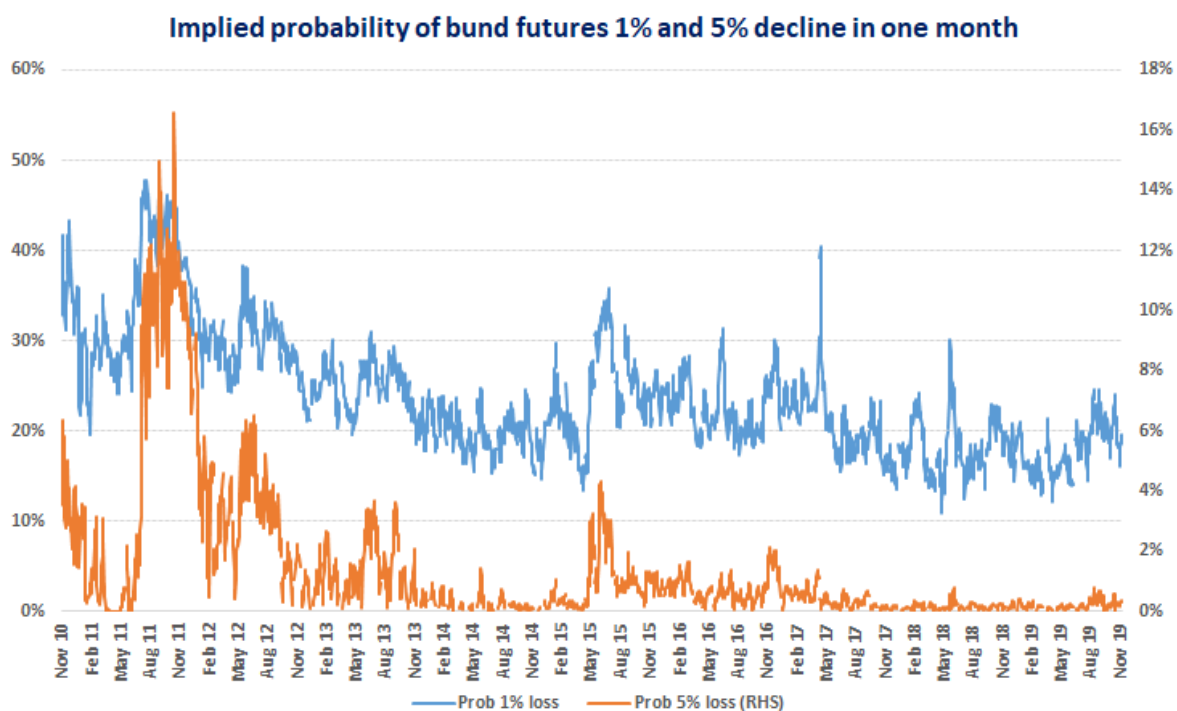


Figure 41: Implied probability of Bund futures 1% and 5% price decline in one month. *Source: Bloomberg, authors' calculations.*

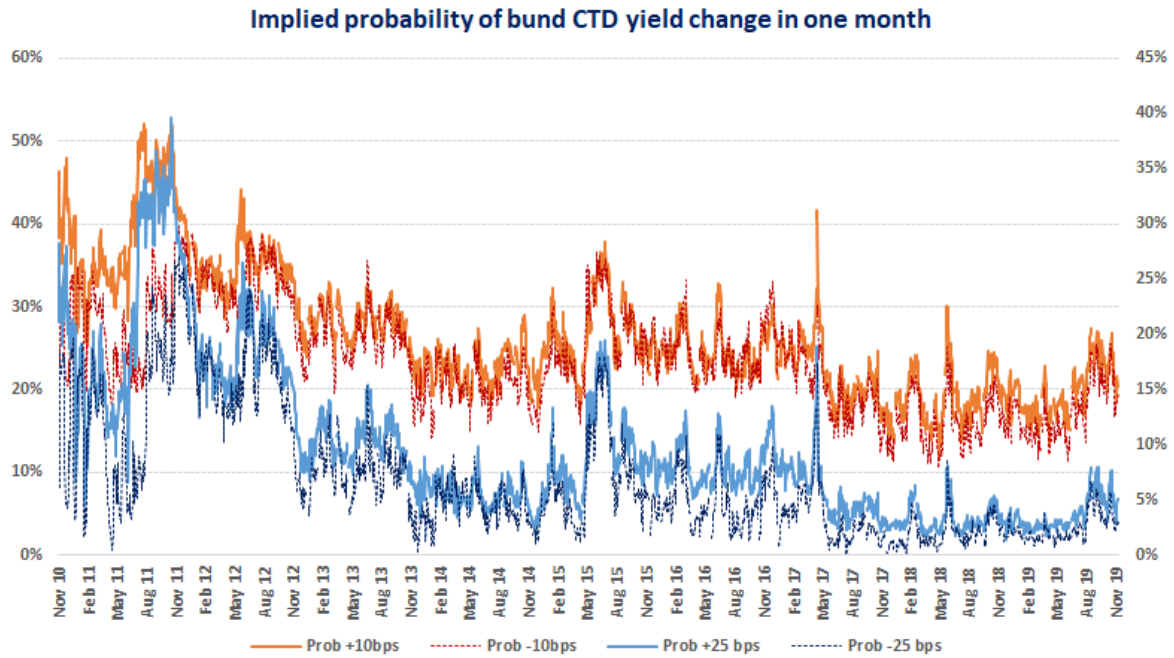


Figure 42: Implied probability of 10 (orange) and 25 (blue) basis points change in yields in CTD bonds of Bund futures. Solid (dashed) line shows the probability of yield increase (decrease). *Source: Bloomberg, authors' calculations.*

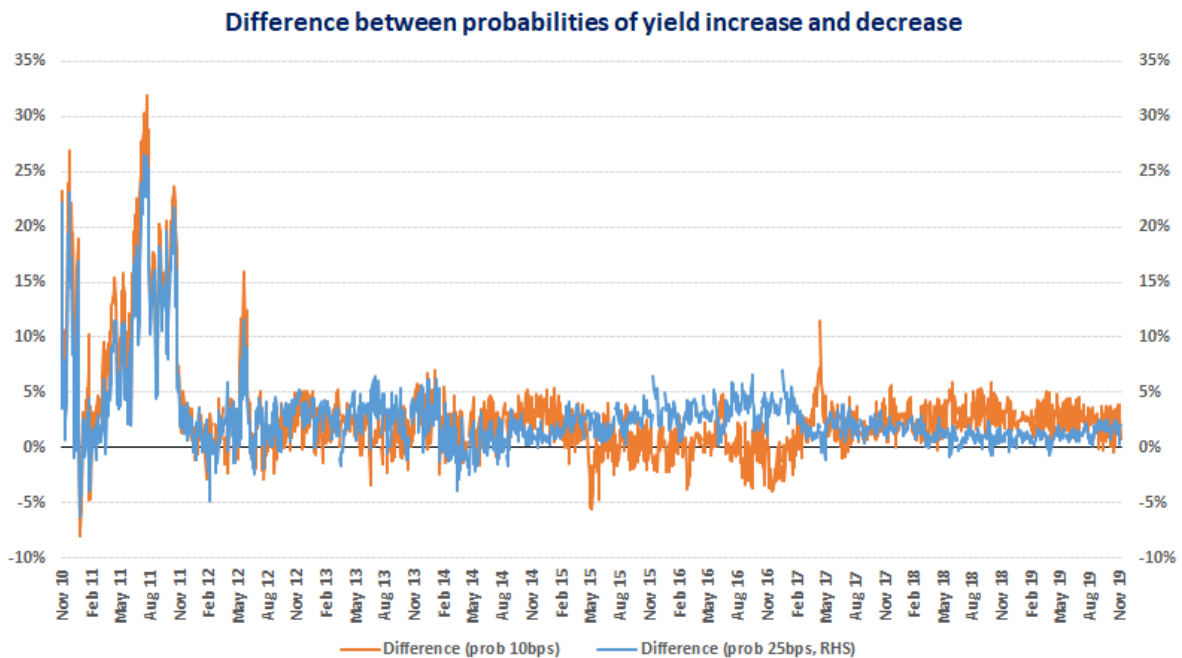


Figure 43: Probability of 10bps (LHS) and 25bps (RHS) yield increase of Bund futures CTD minus the probability of 10bps and 25bps decrease. *Source: Bloomberg, authors' calculations.*

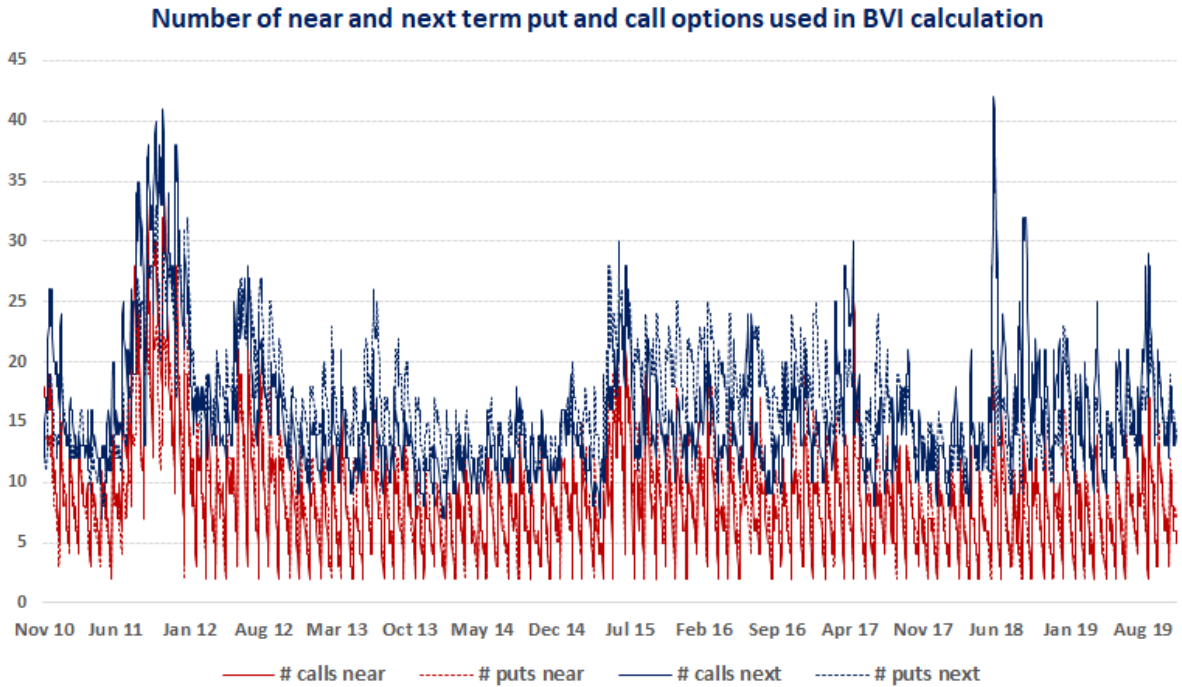


Figure 44: Number of near and next term call and put options used in Bund volatility index calculation. On average, 9 call and 9 put options are used in calculation of the near-term variance and 15 calls and 16 puts are used in calculation of next term variance in our full sample. *Source: Bloomberg, authors' calculations.*

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