ESM BRIEFS NO. 3 - TECHNICAL APPENDIX

A. ECONOMETRIC STRATEGY

Our econometric strategy is aimed at identifying a benchmark — or sectoral norm — for evaluating common trends in both labour productivity and employment growth at the sectoral level across 19 Euro Area (EA) countries. This benchmark enables to disentangle trends common to all countries from idiosyncratic sector-specific deviations at the country level:

- The sectoral norm represents the common long-term trend in labour productivity and employment growth for a given sector across all EA countries. It serves as a benchmark that captures shared structural forces.
- The sectoral gap captures the country specific deviation of a sector's performance from the sectoral norm or benchmark. It reflects the combined effect of country-specific structural factors and idiosyncratic shocks and provides a measure of how much a sector is underperforming or outperforming relative to the common sectoral norm.

B. METHODOLOGY

Low band-pass filter

We use data for real Gross Value Added (GVA) and employment, measured as hours worked, at the sectoral level for 19 EA countries[1] from 1995 to 2023. The long-run evolution of the data is captured by using the Christiano and Fitzgerald (2003) band-pass filter with a periodicity of 11 years. Using filtered series, GVA growth at the country level is first decomposed in the sum of growth in labour productivity[2], lp_t, and hours worked, h_t:

$$\frac{\Delta GVA_t}{GVA_{t-1}} \cong \frac{\Delta lp_t}{lp_{t-1}} + \frac{\Delta h_t}{h_{t-1}} \ .$$

Labour productivity growth is then decomposed into the contribution coming from labour productivity growth at the sectoral level (within-sector effect) and aggregate labour productivity gains coming from the relocation of workers towards relatively more productive sectors (relocation effect), i.e.

$$\frac{\Delta lp_t}{lp_{t-1}} = \underbrace{\sum\left(\frac{GVA^i_{t-1}}{GVA_{t-1}} \times \frac{\Delta lp^i_t}{lp^i_{t-1}}\right)}_{Within-Sector\ Growth} + \underbrace{\sum\left(\frac{lp^i_t}{lp_{t-1}} \times \Delta\alpha^i_t\right)}_{Between-Sector\ Growth}$$

where we define α^i_t as the labour share of sector *i*. In turn, the growth in hours worked is decomposed as the sum of sectoral growth according to:

$$\frac{\Delta h_t}{h_{t-1}} = \sum\left(\alpha^i_t \times \frac{\Delta h^i_t}{h^i_{t-1}}\right)$$

[1] Malta and Ireland are excluded from the estimation but included in the analysis due to a lack of pre-2000 real-term data for Malta and distortions in Ireland's GDP and GVA caused by the impact of foreign-owned multinational enterprises. Croatia is not considered in either the estimation or the analysis.

[2] Labour productivity is defined as the ratio of GVA, and total hours worked.

Unobserved Components Model

We then use an unobserved component model to decompose country- and sector-specific labour productivity and employment growth into a sector-specific trend common to all EA countries, γ_t^i, a country-specific trend common to all sector within each country, μ_t^c, and a country-sector-specific residual, ϵ_t^{ic}, according to the following specification:

$$\frac{\Delta a_t^{ic}}{a_{t-1}^{ic}} = \gamma_{a,t}^i + \mu_{a,t}^c + \epsilon_{a,t}^{ic}$$
$$\mu_{a,t}^c = \mu_{a,t-1}^c + \eta_{a,t}^c$$
$$\gamma_{a,t}^i = \gamma_{a,t-1}^i + e_{a,t}^i$$

where $a = \{lp, h\}$. The unobserved components model is estimated in a state-space framework using the Expectation-Maximization (EM) algorithm, with the Kalman filter and smoother employed to extract the latent components. This approach efficiently handles the high-dimensional panel structure while ensuring optimal inference on time-varying trends. To enhance model stability and improve identification, we treat the Euro Area (EA) as an additional "country" in the panel and normalize its country-specific effect to follow a white noise process, i.e.,

$$\mu_{a,t}^{EA} = \eta_{a,t}$$

This constraint anchors the estimation of common sectoral trends and reduces the risk of confounding them with persistent EA-wide shocks, thereby improving the precision of the extracted sectoral norms.

To enhance the stability and convergence of the EM algorithm in estimating the model, we adopt two complementary strategies. First, we regularize the updated covariance matrices, Q and R, by adding a small positive constant, $\tau = 1^{-6}$, to their diagonals. This ensures that both the process and observation noise covariance matrices remain positive definite and numerically well-conditioned, which is crucial for the Kalman filter and smoother. Second, we implement a weighted update scheme, where the new estimates are blended with the previous ones using a smoothing parameter, $\lambda = 0.1$. This approach dampens abrupt changes across EM iterations, reducing the risk of oscillations or divergence, and leads to more stable convergence of the model parameters. These strategies act as a safeguard, without meaningfully distorting the estimates of the underlying structural trends.

Norm-Gap Decomposition

From the Unobserved Components Model, we recover the latent factors, $\hat{\gamma}_{a,t}^i$, as the *sectoral norms*, capturing the common trend across all countries within sector i. The sum $\hat{\mu}_{a,t}^c + \hat{\epsilon}_{a,t}^{ic}$ as the *sectoral gap*, reflecting the country- and sector-specific deviation from this norm. This yields the following decomposition:

$$\frac{\Delta lp_t}{lp_{t-1}} = \underbrace{\sum\left(\frac{GVA_{t-1}^i}{GVA_{t-1}} \times \hat{\gamma}_{lp,t}^i\right)}_{Within-Sector\ Growth\ Norm} + \underbrace{\sum\left(\frac{GVA_{t-1}^i}{GVA_{t-1}} \times (\hat{\mu}_{lp,t}^i + \hat{\epsilon}_{lp,t}^{ic})\right)}_{Within-Sector\ Growth\ Gap} + \underbrace{\sum\left(\frac{lp_t^i}{lp_{t-1}} \times \Delta\alpha_t^i\right)}_{Between-Sector\ Growth}$$

$$\frac{\Delta h_t}{h_{t-1}} = \underbrace{\sum\left(\alpha_t^i \times \hat{\gamma}_{h,t}^i\right)}_{Within-Sector\ Growth\ Norm} + \underbrace{\sum\left(\alpha_t^i \times (\hat{\mu}_{h,t}^i + \hat{\epsilon}_{h,t}^{ic})\right)}_{Within-Sector\ Growth\ Gap}$$