

# Monetary policy, firm heterogeneity, and the distribution of investment rates

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This paper studies the role of lumpy investment for the transmission of monetary policy, its impact on aggregate investment, and the heterogeneous effects across young and old firms.



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**Keywords:** Investment Rate Distribution, Adjustment Costs, Lumpy Investment, Heterogeneous Sensitivity, Extensive Margin, Monetary Policy

**JEL codes:** E52, E22, D21, D22

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# Monetary policy, firm heterogeneity, and the distribution of investment rates\*

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May 12, 2024

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# 1 Introduction

Understanding the investment channel of monetary policy is important for policymakers because investment is a sizable and the most volatile component of aggregate GDP. To this end, the literature has extensively studied the effect of monetary policy on the *average* investment rate.<sup>1</sup> However, an estimated effect on the average investment rate can reflect a *parallel shifting* of the entire distribution or a change in the *shape* of the distribution. How does monetary policy affect the distribution of investment rates? Which part of the distribution is responsible for the change in the average investment rate? Moreover, a growing academic literature documents heterogeneous effects of monetary policy on the investment rates of different groups of firms, see, e.g., [Gertler and Gilchrist \(1994\)](#), [Ottonello and Winberry \(2020\)](#), [Jeenas \(2023\)](#), and [Cloyne et al. \(2023\)](#).<sup>2</sup> Which part of the distribution drives these heterogeneous effects on *average* investment rates? The answers to these questions are important to understand the transmission of monetary policy. In particular, they can shed light on the frictions that matter for the (heterogeneous) effects of monetary policy on firm investment.

In this paper, we study the investment channel of monetary policy, while paying special attention to the distribution of investment rates. We provide three main pieces of evidence. First, monetary policy changes the *shape* of the distribution of investment rates. Specifically, an expansionary monetary policy shock leads to fewer small and zero investment rates and more large investment rates. Second, the change in the shape of the investment rate distribution is more pronounced among young firms than among old firms. These findings highlight the relevance of the extensive margin investment decision—whether to invest or not—for the transmission of monetary policy. Third, a decomposition exercise reveals that the extensive margin accounts for around 50% of the effect of monetary policy on the *average* investment rate and for more than 50% of the *heterogeneous effect* on firms of different age groups.

We develop a heterogeneous-firm model that combines capital adjustment costs, firm entry and exit, and nominal rigidities to rationalise our empirical findings. The presence of fixed adjustment costs gives rise to lumpy investment behaviour and an investment channel of monetary policy along the extensive margin. That is, an interest rate cut induces some firms to switch from not investing to making a sizable investment. Therefore, monetary policy reshapes the distribution of investment rates. In

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<sup>1</sup>See, for example, [Ottonello and Winberry \(2020\)](#), [Jeenas \(2023\)](#), or [Cloyne et al. \(2023\)](#).

<sup>2</sup>[Cloyne et al. \(2023\)](#) document that investment rates of young firms are on average more sensitive to monetary policy than those of old firms. [Gertler and Gilchrist \(1994\)](#) show a similar result for small and large firms. Clearly, these findings are connected, as age and size are strongly correlated in the data. In this paper, we focus on age but emphasise and show that our results are similar when comparing small and large firms.

addition, the model generates heterogeneous effects of monetary policy on the average investment rates of young and old firms. The main reason is that young firms can more easily be induced to make an investment because many young firms are far away from their optimal size while many old firms are close to it. The calibrated model attributes more than 50% of the heterogeneous effect across age groups to the extensive margin, as in the data. Finally, the model implies that the investment channel of monetary policy has weakened by around 12% due to the secular decline in firm dynamism and is stronger in booms than in recessions.

In more detail, we study the investment channel using quarterly firm-level investment data from Compustat in combination with identified monetary policy shocks as in [Ottonello and Winberry \(2020\)](#) and [Cloyne et al. \(2023\)](#). In contrast to the existing literature, we estimate the effects of monetary policy on quantiles and bins of the investment rate distribution rather than solely focusing on the first moment of the distribution—the average investment rate. We uncover that the upper quantiles respond substantially more to a monetary policy shock than the lower quantiles do. This shows that monetary policy changes the shape of the distribution of investment rates. To further investigate the change in the distribution, we estimate the effects of monetary policy on the *fraction* of firms in each bin of the investment rate distribution. Comparing the binned distributions before and after an expansionary monetary policy shock, we illustrate that fewer firms make small or no investments and more firms make large investments—**Fact 1**. This novel evidence suggests the presence of a quantitatively relevant investment channel of monetary policy along the extensive margin.

Conducting the same empirical analysis for young and old firms separately, we uncover that the effect of monetary policy on the shape of the distribution of investment rates is more pronounced among young firms than among old firms—**Fact 2**. This finding suggests that the extensive margin investment channel is particularly important for young firms. We substantiate this view by estimating the effects of monetary policy on the *spike rate*, defined as the fraction of firms whose quarterly investment rate exceeds 10%, and on the *inaction rate*, defined as the fraction of firms whose quarterly investment rate is smaller than 0.5% in absolute value. The spike rate rises and the inaction rate drops more strongly for young firms than for old firms, corroborating the interpretation that monetary policy induces more young than old firms to switch from being inactive to making a sizable investment.

Closely related to our findings, the empirical literature has documented that young firms' *average* investment rates are more responsive to monetary policy than old firms' ([Cloyne et al., 2023](#)). Differences in the responsiveness of small and large firms—as

already shown in [Gertler and Gilchrist \(1994\)](#)—are also significant but quantitatively less pronounced. Conventional wisdom views these findings as supporting the financial accelerator mechanism, based on the narrative that young firms are financially constrained<sup>3</sup> and monetary policy affects financial conditions. The novel empirical evidence presented in this paper suggests that besides financial acceleration, the extensive margin investment decision, arising from fixed adjustment costs, is important for the heterogeneous effects on young and old firms, too. We provide additional evidence to corroborate that the heterogeneous effect of monetary policy on the investment rate distributions does not reflect financial frictions. We show that even among firms deemed unlikely to be financially constrained—characterised by low leverage, high liquidity, or having paid dividends—the spike and inaction rates of young firms remain more responsive to monetary policy than those of older firms.

A final empirical exercise quantifies the relative importance of the extensive margin. We decompose the effect of monetary policy on the average investment rate into contributions arising from the extensive and intensive margin, respectively. We use the change in the spike rate to proxy for the extensive margin. Our decomposition reveals that the extensive margin accounts for around 50% of the effect of monetary policy on the average investment rate. In addition, more than 50% of the heterogeneous effect on young and old firms' average investment rates is due to the extensive margin.

The second part of the paper interprets the empirical findings through the lens of a general equilibrium heterogeneous-firm model with capital adjustment costs, firm life cycle dynamics, and nominal rigidities. Fixed capital adjustment costs give rise to lumpy investment behaviour at the firm level and an investment channel of monetary policy along the extensive margin. As a result, monetary policy affects the distribution of investment rates, consistent with the first empirical finding.

Moreover, the model illustrates that the presence of an extensive margin investment decision creates *heterogeneous* effects on investment rate distributions, as well as average investment rates, of young and old firms, consistent with the second empirical finding. Entry and exit give rise to realistic firm life cycle profiles and an age distribution. The age-group-specific average investment rate is the fraction of investing firms (hazard rate) times the investment rate conditional on investing. The heterogeneous effect on different age groups along the extensive margin arises from two channels.

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<sup>3</sup>[Rauh \(2006\)](#), [Fee et al. \(2009\)](#), [Hadlock and Pierce \(2010\)](#), and more recently [Cloyne et al. \(2023\)](#) argue that young firms are more likely financially constrained than old firms. [Gertler and Gilchrist \(1994\)](#) rely on the narrative that "...the costs of external finance apply mainly to younger firms, firms with a high degree of idiosyncratic risk, and firms that are not well collateralized. These are, on average, smaller firms..." to motivate the use of firm size as a proxy for financial frictions.

First, monetary policy has a heterogeneous effect on hazard rates. More specifically, an interest rate cut induces more young than old firms to switch from inaction to making an investment. The reason is that young firms are on average far away from their optimal level of capital and have a high marginal product of capital, and can therefore relatively easily be induced to make an investment. In contrast, many old firms are close to their optimal size and cannot easily be induced to invest. Second, even without a heterogeneous effect on hazard rates, we would observe a higher *average* effect on young firms. This is because young firms choose a higher investment rate conditional on investing as they are, again, on average farther away from their optimal level of capital.

When the model is calibrated to aggregate and firm-level data from the United States, young firms are almost twice as responsive to a monetary policy shock as old firms, explaining a large portion of the heterogeneous effect documented empirically. In sum, the model not only provides a novel explanation for existing empirical evidence, such as the heterogeneous effects of monetary policy on average investment rates of young and old firms, but also sheds light on newly uncovered evidence in this paper, specifically the heterogeneous effects on the distribution of investment rates.

Finally, we explore the aggregate implications of the heterogeneous-firm model with fixed adjustment costs and firm life cycle dynamics. Broadly speaking, the model implicates that monetary policy is particularly effective whenever there are many firms that can easily be induced to make a large investment. We provide two examples to emphasise that both long-run trends and cyclical developments are quantitatively relevant. First, we show that the decline in firm dynamism and ensuing “aging” of the firm distribution (i.e., lower share of young firms) observed since the 1980s has, according to the model, made monetary policy about 12% less effective in stimulating investment. Second, monetary policy is less effective in a recession than in a boom, because in a recession, fewer firms are interested in making an investment.

The second example highlights that understanding the frictions underlying firms’ (heterogeneous) investment responses to monetary policy is crucial for guiding macroeconomic policy over the business cycle. We have presented empirical evidence and a theoretical model that emphasise fixed adjustment costs and the investment channel of monetary policy along the *extensive margin*. This closely relates to the literature that highlights the relevance of financial frictions for the investment channel along the *intensive margin*. Both of these two frictions can explain heterogeneous effects on the average investment rate among groups of firms, but they make opposing predictions about the effectiveness of monetary policy in recessions. Financial conditions are typically tighter in recessions, which are therefore associated with a stronger finan-

cial accelerator mechanism.<sup>4</sup> Therefore, if financial frictions are more critical for firms' investment decisions, one would expect monetary and fiscal policy to be more effective in recessions. Conversely, if lumpy investment behaviour due to the presence of fixed adjustment costs is more critical, recessions are characterised by fewer firms being at the margin of investing. This makes macroeconomic policies less effective (see also [Winberry 2021](#)). The relevance of lumpy investment and fixed adjustment costs is consistent with separate evidence uncovered in the empirical literature, indicating that monetary and fiscal policy interventions are less potent in recessions ([Tenreyro and Thwaites, 2016](#); [Ramey and Zubairy, 2018](#)).

**Literature Review** First and foremost, this paper relates to the empirical literature studying the investment channel of monetary policy using aggregate data (e.g., [Christiano et al. 2005](#)), and firm-level data (e.g., [Gertler and Gilchrist 1994](#), [Ottonello and Winberry 2020](#), [Jeenas 2023](#), [Cloyne et al. 2023](#), [Jungherr et al. 2022](#), [González et al. 2022](#), and [Asriyan et al. 2022](#)). So far, this literature has focused on the effects on aggregate investment or average investment rates. Our contribution is to document how monetary policy affects the entire distribution of investment rates as well as moments thereof, such as the spike rate and the inaction rate. A strand of this literature studies the heterogeneous effects across various groups of firms. We contribute to this strand by showing that between young and old firms, not only the average effects differ, but also the effects on the distribution as well as on spike and inaction rates. This shows that for understanding heterogeneous effects across groups of firms, not only financial frictions but also real frictions are important to consider. Both of these empirical findings are also important to understand how the aggregate investment channel is shaped by two features of firm-level investment behaviour—lumpy investment and life cycle patterns.

The empirical findings also relate to the empirical literature studying lumpy investment. This literature has mainly produced two types of evidence. First, the *unconditional* distribution of firm-level investment rates is in line with the presence of fixed adjustment costs (e.g., [Caballero et al. 1995](#), [Cooper et al. 1999](#), and [Cooper and Haltiwanger 2006](#)). Second, the behaviour of *aggregate* investment in response to macroeconomic shocks is in line with the presence of fixed adjustment costs (e.g., [Caballero and Engel 1999](#), [Bachmann et al. 2013](#), and [Fang 2023](#)). In addition, [Gourio and Kashyap \(2007\)](#) estimate the cyclicity of the spike rate of firms' investments. Our contribution

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<sup>4</sup>Even though the capital adjustment costs that we impose in our model can be interpreted as stand-ins for financial frictions, the model does not feature a financial accelerator mechanism. This is because, by construction, the capital adjustment costs are themselves not affected by aggregate shocks, including monetary policy shocks.



to this literature is to demonstrate the relevance of lumpy investment for the transmission of monetary policy, thereby contributing to the literature by establishing the importance of the micro-level lumpiness of investment for macroeconomic dynamics. Specifically, we first of all document the movements of the *entire distribution* of investment rates, including moments thereof like spike and inaction rates, *conditional* on monetary policy shocks. Second, we show that there is an important interaction between firm life cycles and lumpy investment as we find heterogeneous effects across age groups along the extensive margin.<sup>5</sup> Hence, our evidence shows that lumpy investment behaviour is important for understanding of both the cross-sectional and the aggregate effects of monetary policy.

Finally, our paper contributes to the theoretical and quantitative literature on the relevance of the extensive margin investment decision and lumpy investment behaviour at the firm level for aggregate investment, particularly for its responsiveness to shocks over the business cycle. Important contributions include Caballero et al. (1995), Caballero and Engel (1999), Thomas (2002), Khan and Thomas (2003), Khan and Thomas (2008), Bachmann et al. (2013), House (2014), Koby and Wolf (2020), Winberry (2021), and Baley and Blanco (2021). Monetary policy shocks in models with fixed adjustment costs have been analysed by Reiter et al. (2013), Reiter et al. (2020), and Fang (2023). We contribute to this line of research by incorporating firm life cycles into an otherwise standard heterogeneous-firm model with lumpy investment. The combination of these two features is important for two reasons. First, introducing firm life cycles allows us to examine heterogeneous cross-sectional effects of monetary policy across firms of different age groups, which is necessary to rationalise our empirical findings. Second, the combination of lumpy investment and life cycles is not only important for the cross-section but also for the *aggregate* investment channel of monetary policy. Specifically, we quantify the weakening of the investment channel of monetary policy due to firm aging.

The remainder of this paper is organised as follows. Section 2 presents our empirical results. Section 3 describes the New Keynesian heterogeneous-firm model. Section 4 calibrates the model and analyses the effects of monetary policy. Section 5 concludes.

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<sup>5</sup>In contemporaneous work, Lee (2023) estimates the effect of monetary policy shocks on the spike rates of small and large firms. We investigate the effect on the entire distribution of investment rates of young and old firms, in addition to spike rates. Lee (2023) uses the estimates by firm size to calibrate a real business cycle model with size-dependent fixed adjustment costs and aggregate TFP shocks. We rationalise our findings in a New Keynesian sticky-price model with firm life cycles and derive aggregate implications.

## 2 Empirical Evidence

We present three pieces of evidence that are important to understand the investment channel of monetary policy. Section 2.1 introduces the data used throughout the analysis. Section 2.2 describes the local projection model used to estimate impulse response functions (IRFs). Section 2.3 documents the effects of monetary policy on the distribution of investment rates. Section 2.4 presents the heterogeneous effects of monetary policy by firm age. Section 2.5 decomposes the (heterogeneous) effects of monetary policy into contributions arising from the extensive and intensive margins, respectively.

### 2.1 Firm-Level Data

We use quarterly firm-level data from Compustat. Our sample begins with 1986Q1 and ends with 2018Q4. We exclude firms located outside the United States, with incomplete or questionable information (e.g. negative reported sales), and those not suitable for our analysis (e.g. financial firms) from the sample. Details on the sample selection are relegated to Appendix D.1. Since information on firm age in Compustat is scarce, we merge age information from WorldScope and Jay Ritter’s database, as explained in Appendix D.2.

Capital stocks reported in Compustat are *accounting* capital stocks and do not perfectly reflect *economic* capital stocks.<sup>6</sup> To address this issue, we use a Perpetual Inventory Method (PIM) to compute real economic capital stocks, building on [Bachmann and Bayer \(2014\)](#). Details of this procedure are explained in Appendix D.3. Our baseline measure of the investment rate is  $i_{jt} = \frac{CAPX_{jt} - SPPE_{jt}}{INVDEF_t \times k_{jt-1}}$ , thus, real capital expenditures (CAPX) net of sales of capital (SPPE) divided by the lagged real economic capital stock ( $k$ ). More details on the construction of variables are given in Appendix D.4.

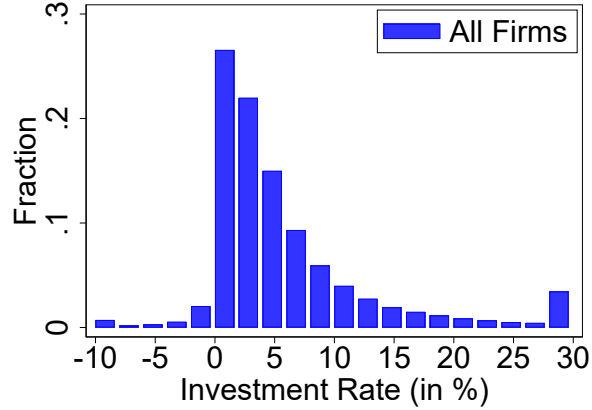
For parts of the subsequent analysis, we aggregate the firm-level data to quarterly investment rate distributions and moments thereof.<sup>7</sup> The distribution of investment rates, shown in Figure 1, depicts some well-known features of investment rate distributions. That is, the distribution has a positive skewness, a long right tail, substantial mass at 0, and very few negative observations.

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<sup>6</sup>On the one hand, accounting depreciation is driven by tax incentives and usually exceeds economic depreciation. On the other hand, accounting capital stocks are reported at historical prices, not current prices. With positive inflation, both issues make economic capital stocks exceed accounting capital stocks.

<sup>7</sup>Moments which are sensitive to outliers, such as the mean, are winsorized. Importantly, winsorizing is done by group and quarter. This ensures that the winsorizing process does not systematically bias our sample.

**Figure 1:** Distribution of Investment Rates



*Notes:* This figure plots the distribution of quarterly investment rates of firms in Compustat. The investment rate is real capital expenditures (CAPX) net of sales of capital (SPPE) divided by the lagged real economic capital stock. Sample: 1986Q1 - 2018Q4.

## 2.2 Local Projections

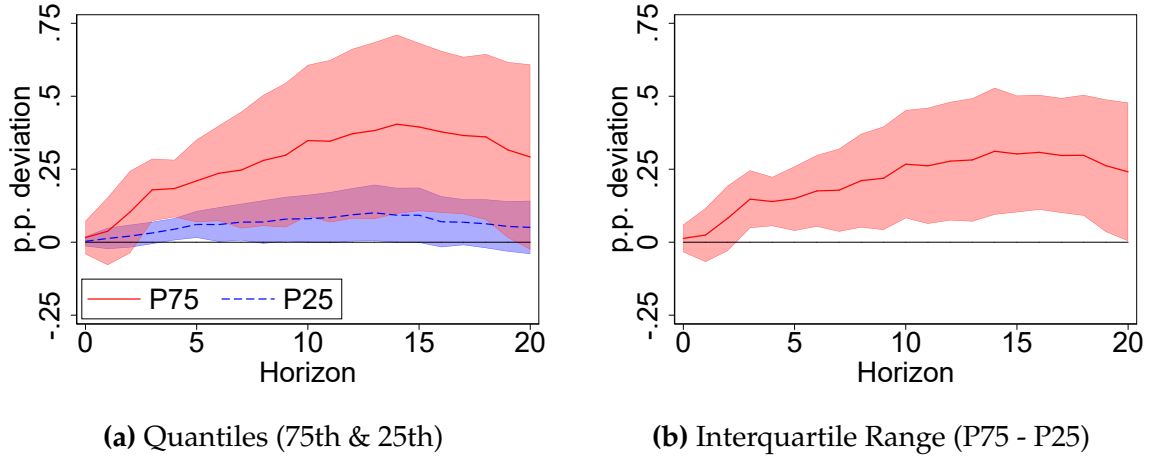
To estimate the effects of monetary policy shocks, we estimate the following simple local projection (LP) models:

$$y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h} \quad (1)$$

where  $y_t$  indicates the outcome variable,  $\epsilon_t^{MP}$  is the monetary policy shock,  $q_t$  is the calendar quarter, and  $\mathbb{1}\{q_{t+h} = j\}$  are quarter dummies that are included to address seasonality. We use the monetary policy shocks implied by the Proxy SVAR in [Gertler and Karadi \(2015\)](#). These are extracted after updating the time series data used in the VAR as well as the high-frequency instruments. Details are relegated to [Appendix D.5](#). Unless stated otherwise, the shocks are scaled to reduce the 1-year Treasury yield on impact by 25 basis points. Throughout, we use Newey-West standard errors to account for heteroskedasticity and autocorrelation.

Before turning to our novel findings, we verify that the monetary policy shocks have plausible effects on aggregate variables. We show in [Appendix D.6](#) that an expansionary shock leads to hump-shaped increases in both investment and real GDP. The peak effects are 1.4% (investment) and 0.35% (real GDP), respectively.

**Figure 2:** Effect of Monetary Policy on Quantiles of the Investment Rate Distribution



*Notes:* This figure plots the effect of a monetary policy shock on statistics of the investment rate distribution. The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

### 2.3 Fact 1: Shape of the Distribution of Investment Rates

The literature has extensively studied the effect of monetary policy on the *average* investment rate.<sup>8</sup> On the one hand, this estimated effect could reflect that all firms increase their investment rate by the same (average) amount. In this case, we would expect the distribution of investment rates to *shift* to the right, but not change its shape. On the other hand, the change in the average investment rate could reflect that only a small number of firms increase their investment rate, but by a large amount. In this case, we would expect a change in the *shape* of the distribution of investment rates.

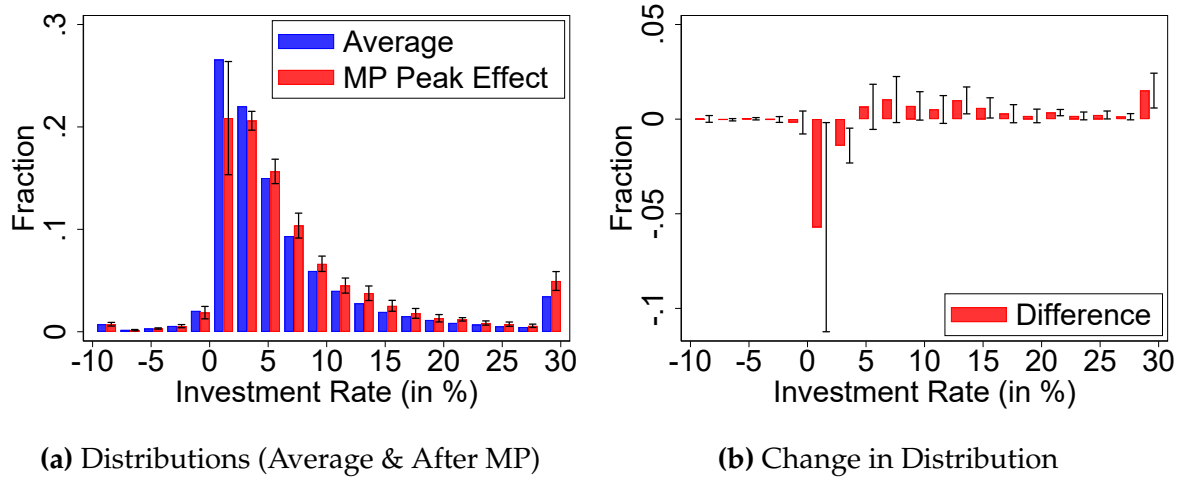
To investigate whether monetary policy affects the distribution of firm-level investment rates, we estimate the effects on different quantiles of the investment rate distribution. This is done by using the time series of the respective quantiles of the distribution as outcome variables in the empirical model (Equation 1).<sup>9</sup> If the increase in the average investment rate reflects a parallel shifting of the distribution, the effect on all quantiles must be identical.

Figure 2 shows the effect of a monetary policy shock on selected quantiles of the investment rate distribution. Panel (a) plots the responses of the 25th (blue line) and the 75th (red line) percentiles. It is evident that the right tail of the investment rate distribution (the 75th percentile) responds more strongly than the left tail (the 25th

<sup>8</sup>We show the effect of monetary policy on the average investment rate in Panel (a) of Figure 7.

<sup>9</sup>Loria et al. (2023) have recently applied a similar two-step quantile local projection approach to estimate the effects of macroeconomic shocks on the conditional quantiles of GDP growth.

**Figure 3:** Effect of Monetary Policy on the Distribution of Investment Rates



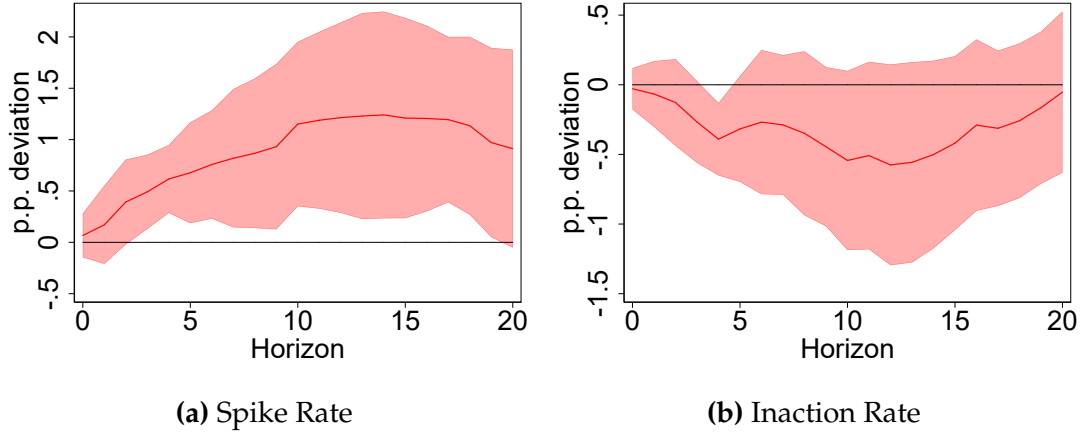
*Notes:* Panel (a) plots the effect of a monetary policy shock on bins of the investment rate distribution. Blue bars depict the average distribution, red bars depict the predicted distribution at horizon 13 (peak effect) after a monetary policy shock. Panel (b) plots the difference between the bars in panel (a). Black lines indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. To improve readability, the shocks are scaled to reduce the 1-year Treasury yield by 100 basis points.

percentile). At the peak, the 75th percentile of the investment rate distribution rises by 40 basis points, while the 25th percentile rises by only 10 basis points. This difference is statistically significant, as illustrated by the IRF of the corresponding interquartile range (Panel b). These findings are robust to the alternative choices of quantiles as shown in Figure A.1. The disproportionate change in the right tail compared to the left tail shows that monetary policy changes the shape of the investment rate distribution.

To further investigate how monetary policy affects the distribution of investment rates, we use the binned distribution depicted in Figure 1 and regress the time series of the *fraction* of firms in each bin on the monetary policy shock. Panel (a) of Figure 3 shows the average distribution of investment rates (blue bars) next to the predicted distribution at the horizon at which the effect of the monetary policy shock peaks (red bars).<sup>10</sup> Panel (b) illustrates the difference between the two distributions. Confirming the evidence from Figure 2, there is a visible change in the distribution of investment rates. In particular, after an expansionary monetary policy shock, there are fewer small investment rates and more large investment rates. The fraction of firms in the bins [0, 2] and [2, 4] falls significantly, while the fraction of firms in all other positive bins rises, most sizably and significantly in the bin with the largest investment rates [28,  $\infty$ ). In contrast, the share and distribution of negative investment rates are not meaningfully

<sup>10</sup>Horizon 13 is when the peak effect on the average investment rate is reached.

**Figure 4:** Effect of Monetary Policy on the Spike and Inaction Rate



*Notes:* This figure plots the effect of a monetary policy shock on the spike rate and the inaction rate of all firms. A spike is an investment rate exceeding 10%, inaction is an investment rate less than 0.5% in absolute value. The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

affected. This suggests that the effect of monetary policy on the *average* investment rate is driven to a sizable degree by the extensive margin, i.e., a small number of firms switch from making a small or no investment to making a large investment.

**Effect on the Spike and Inaction Rate** To further investigate the interpretation that the extensive margin investment decision is important for the effect of monetary policy on firm investment behaviour, we look at two additional statistics of the investment rate distribution. These are the *spike rate*, defined as the fraction of firms whose quarterly investment rate exceeds 10%, and the *inaction rate*, defined as the fraction of firms whose quarterly investment rate is smaller than 0.5% in absolute value.<sup>11</sup> Corroborating our interpretation, we find that following an expansionary monetary policy shock, the inaction rate falls and the spike rate rises, as shown in Figure 4.

<sup>11</sup>In annual data, an investment spike is typically defined as an investment rate above 20%, so about twice the average investment rate, which, in most representative datasets, ranges between 10% and 12% (Zwick and Mahon, 2017). Since we do not use annual, but quarterly data and Compustat features higher average investment rates, as discussed in Appendix D.3, we define an investment spike to be a quarterly investment rate exceeding 10%. This too is an investment rate roughly twice the average investment rate. Inaction is typically defined as an annual investment rate less than 1% in absolute value. For the same reasons as above, we define inaction as a quarterly investment rate smaller than 0.5% in absolute value.

## 2.4 Fact 2: Heterogeneous Effects across Age Groups

Our empirical strategy can also be applied to investigate *group-specific* investment rate distributions. [Cloyne et al. \(2023\)](#) have documented that after an expansionary monetary policy shock, young firms increase their investment rates *on average* by much more than old firms. We replicate this finding in Figure [A.2](#). This difference in average effects could reflect the intensive margin—young firms changing their investment rates by more than old firms—or the extensive margin—more young firms changing their decision whether to invest at all than old firms. To understand the relevance of the extensive margin in explaining the existing evidence, we estimate the effect of monetary policy on age-specific investment rate distributions.

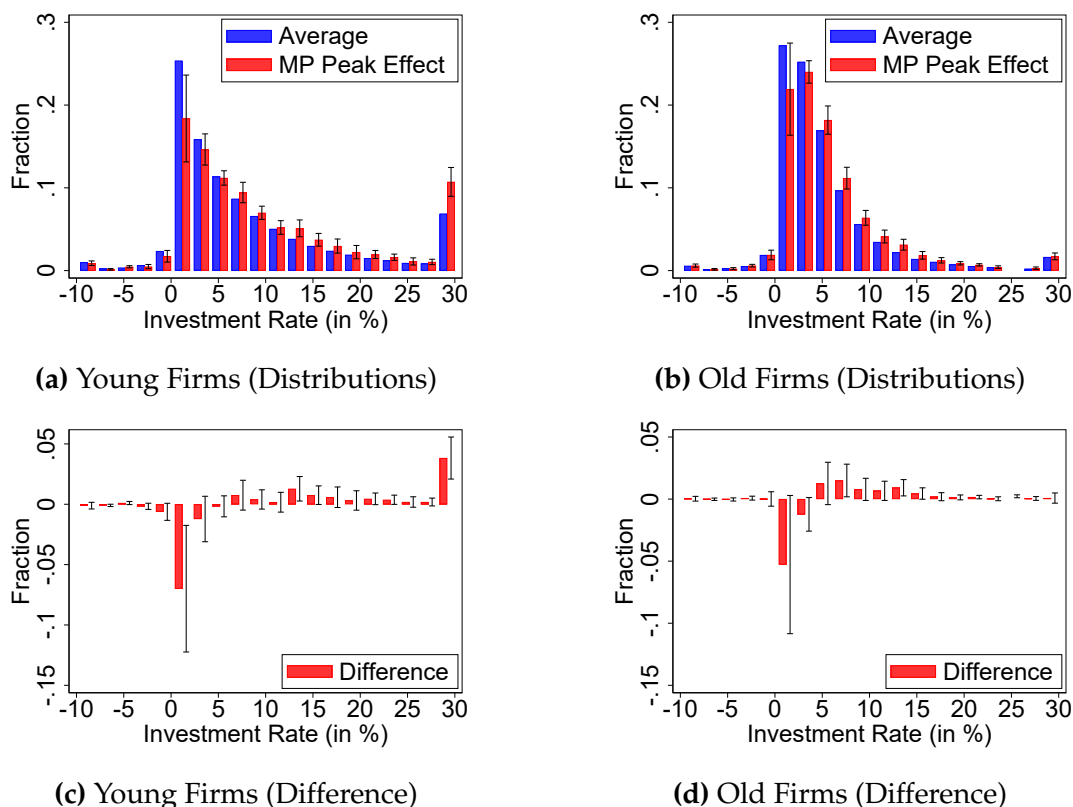
**Heterogeneous Effect on Investment Rate Distributions** Figures [A.3](#) and [A.4](#) show that the disproportionate effects of monetary policy on the right tail of the investment rate distribution (i.e., the upper quantiles), documented for all firms in Figure [2](#), are present among both the group of young firms and the group of old firms. However, these effects are quantitatively much more pronounced among young firms.

We investigate further the effect of monetary policy on age-specific investment rate distributions using binned distributions. Figure [5](#) compares the average distribution of investment rates of young (Panel a) and old (Panel b) firms with the predicted distributions after a monetary policy shock. Panels (c) and (d) plot the differences between the respective distributions. We find that the shape of the distribution changes more sizably and significantly for young firms. In particular, the decrease in zero and small investment rates (bin  $[0, 2]$ ) and the increase in very large investment rates (bin  $[28, \infty)$ ) are more pronounced and statistically significant. This suggests that the extensive margin is important to understand not only the *average* effect of monetary policy on investment rates but also the *heterogeneous* effect across age groups.

**Heterogeneous Effect on Spike and Inaction Rates** To lend further support to the hypothesis that the extensive margin is important for the heterogeneous responsiveness of young and old firms, we look at two additional statistics of the investment rate distribution, namely, the *spike rate* and the *inaction rate*. Figure [6](#) shows that the spike rate rises and the inaction rate drops more strongly for young firms. Both differences are statistically significant.

**Financial Frictions are not driving the Heterogeneous Effects** Firm age is commonly used to proxy for financial constraints. This reflects the hypothesis that access to external finance depends on collateral or reputation, which young firms may lack.

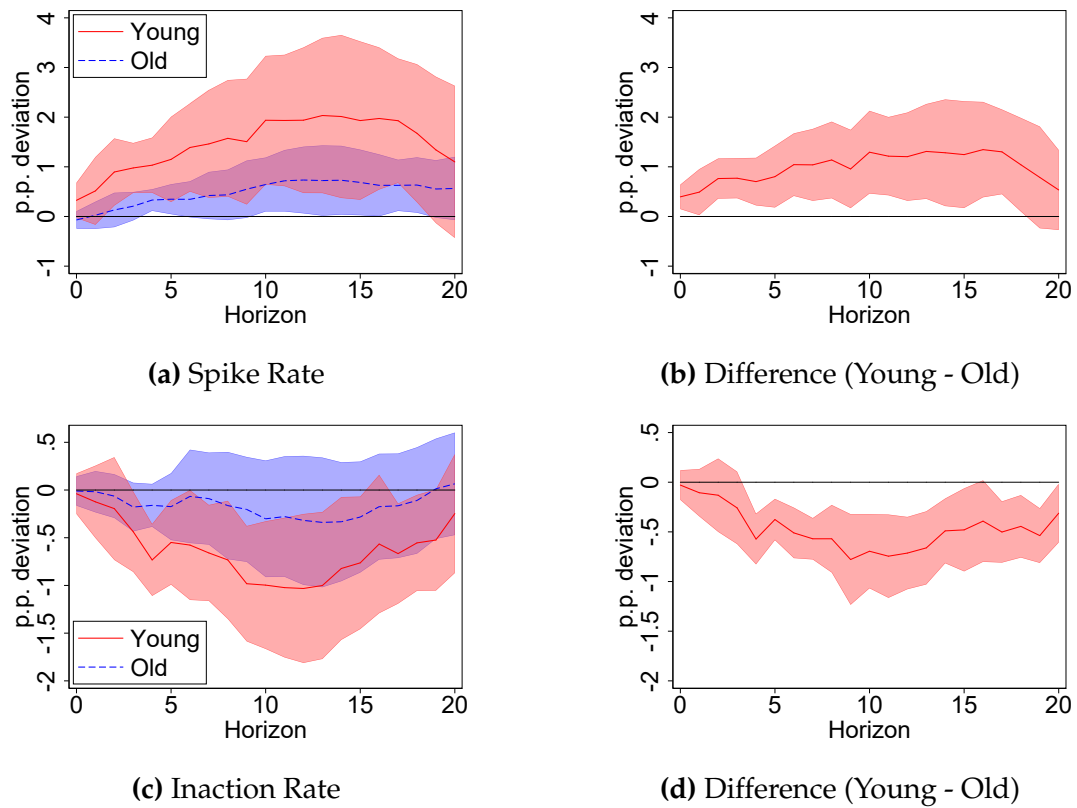
**Figure 5:** Effect of Monetary Policy on Age-Specific Distributions of Investment Rates



*Notes:* Panels (a) and (b) plot the effect of a monetary policy shock on bins of the investment rate distribution for young (a) and old (b) firms. Young (old) firms are less (more) than 15 years old. Blue bars depict the average distribution, red bars depict the predicted distribution at horizon 13 (peak effect) after a monetary policy shock. Panels (c) and (d) plot the difference between the bars in panels (a) and (b). Black lines indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. To improve readability, the shocks are scaled to reduce the 1-year Treasury yield by 100 basis points.



**Figure 6:** Effect of Monetary Policy on Age-Specific Spike & Inaction Rates



*Notes:* This figure plots the effect of a monetary policy shock on the spike rate and the inaction rate of young and old firms. Young (old) firms are less (more) than 15 years old. A spike is an investment rate exceeding 10%, inaction is an investment rate less than 0.5% in absolute value. The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h e_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

Our evidence does not reject this hypothesis but highlights that next to financial constraints, there are additional differences between young and old firms.<sup>12</sup> In particular, young firms have higher average investment rates, higher spike rates (as also shown in Figure 9), and are more likely to change their extensive margin investment decision in response to monetary policy.

To corroborate our argument that the heterogeneous effects of monetary policy do not reflect financial frictions, we replicate the evidence presented in Figure 6 among groups of firms that are unlikely to be financially constrained. Figure A.5 shows that even among firms that have low leverage, high liquidity, or have paid dividends, the

<sup>12</sup>This argument aligns well with [Crouzet and Mehrotra \(2020\)](#) who argue that large firms are less cyclical than small firms because they are better diversified across industries, but not because of financial frictions, and [Farre-Mensa and Ljungqvist \(2016\)](#) who show that firms classified as financially constrained differ from other firms also along non-financial dimensions.

spike and inaction rates of young firms are more responsive to monetary policy shocks than those of old firms.

## 2.5 The Relative Importance of the Extensive Margin

Finally, we perform a decomposition exercise to quantify the relative importance of the intensive and extensive margin. For this purpose, we classify investment rate observations into “spikes” ( $i_{j,t} > 10\%$ , as before) and “normal” investments ( $i_{j,t} \leq 10\%$ ). It follows that the average (potentially group-specific) investment rate in period  $t$  is

$$\bar{i}_t = \psi_t^s i_t^s + (1 - \psi_t^s) i_t^n \quad (2)$$

where  $\psi_t^s$  is the fraction of firms undertaking a “spike” in period  $t$ ,  $i_t^s$  and  $i_t^n$  are the average investment rates conditional on “spike” and “normal”, respectively. Then, the effect of a monetary policy shock on the average investment rate can be decomposed as follows:<sup>13</sup>

$$\frac{\partial \mathbb{E}(\bar{i}_t)}{\partial \epsilon^{MP}} \approx \underbrace{\frac{\partial \mathbb{E}(\psi_t^s)}{\partial \epsilon^{MP}} (\mathbb{E}(i_t^s) - \mathbb{E}(i_t^n))}_{\text{Extensive Margin}} + \underbrace{\mathbb{E}(\psi_t^s) \frac{\partial \mathbb{E}(i_t^s)}{\partial \epsilon^{MP}} + (1 - \mathbb{E}(\psi_t^s)) \frac{\partial \mathbb{E}(i_t^n)}{\partial \epsilon^{MP}}}_{\text{Intensive Margin}} \quad (3)$$

Intuitively, the extensive margin component reflects the change in the average investment rate that results *only* from changes in the spike rate, while the conditional investment rates are held fixed. Vice versa, the intensive margin component reflects the change in the average investment rate that results *only* from changes in the conditional investment rates, while the spike rate is held fixed.

To implement this decomposition, we construct hypothetical average investment rates that would prevail if there were no changes in the extensive margin ( $\bar{i}_t^{int}$ ) or the intensive margin ( $\bar{i}_t^{ext}$ ):

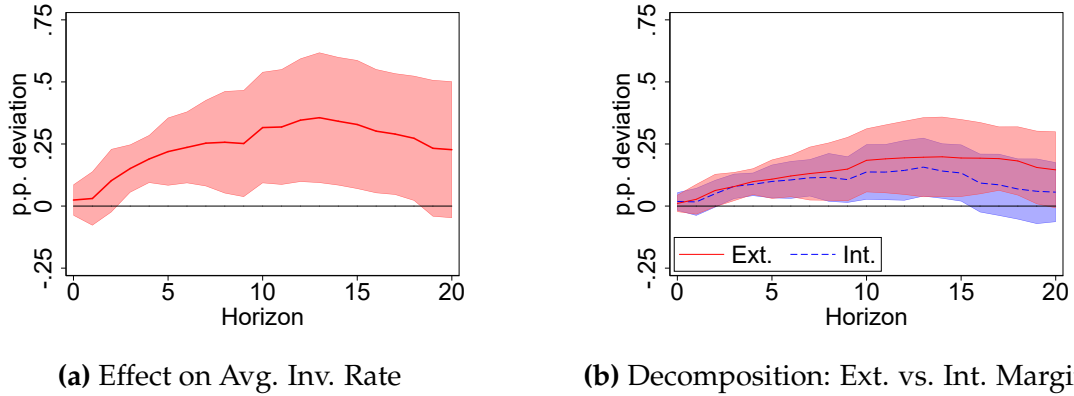
$$\bar{i}_t^{int} = \bar{\psi}^s i_t^s + (1 - \bar{\psi}^s) i_t^n, \quad (4)$$

$$\bar{i}_t^{ext} = \psi_t^s \bar{i}^s + (1 - \psi_t^s) \bar{i}^n. \quad (5)$$

$\bar{i}_t^{int}$  captures fluctuations in the average investment rate arising only from the inten-

<sup>13</sup>This decomposition ignores two covariance terms ( $Cov(\psi_t^s, i_t^s)$ ,  $Cov(\psi_t^s, i_t^n)$ ), which could also be affected by the monetary shock. In the data, their contribution to the total effect on the average investment rate is very small, however. Furthermore, unlike in the model, we cannot perfectly identify “spikes” in the data. Choosing a particular threshold (e.g., 10%) has the drawback that intensive margin adjustments *across* this threshold are captured as extensive margin adjustments. At the same time, extensive margin adjustments with an investment rate below this threshold are captured as intensive margin adjustments.

**Figure 7:** Decomposition of the Effect of Monetary Policy on the Avg. Inv. Rate



Notes: Panel (a) of this figure shows the effect of a monetary policy shock on the average investment rate ( $\bar{i}_t$ ). Panel (b) decomposes this effect into an intensive ( $\bar{i}_t^{int}$ ) and an extensive margin ( $\bar{i}_t^{ext}$ ) contribution, using equation (3). The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

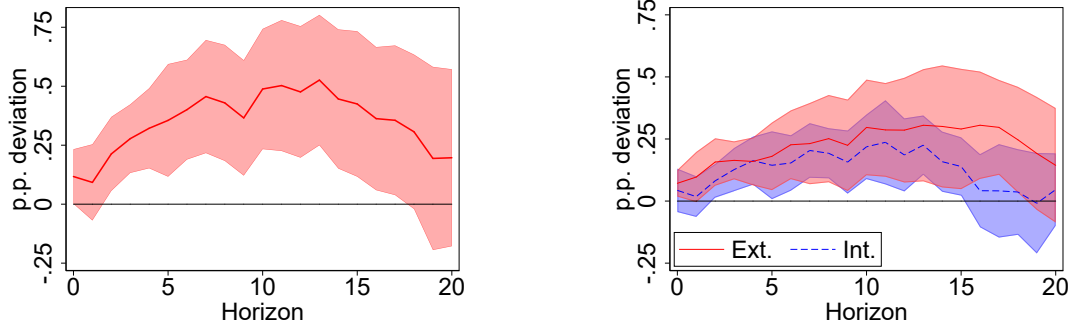
sive margin, because the spike rate,  $\bar{\psi}^s$ , equals its average over time. Vice versa,  $\bar{i}_t^{ext}$  captures fluctuations in the average investment rate arising only from the extensive margin, because the conditional investment rates,  $\bar{i}^n$  and  $\bar{i}^s$ , equal their respective averages over time.

**Decomposition of the Average Effect of Monetary Policy** According to Equation (3), the IRF of the average investment rate ( $\bar{i}_t$ ) is approximately equal to the sum of the IRFs of the two hypothetical investment rates ( $\bar{i}_t^{int}$  and  $\bar{i}_t^{ext}$ ). Figure 7a plots the total effect on the average investment rate and Figure 7b presents the decomposition. It is evident that both margins contribute about 50% to the effect of monetary policy on the average investment rate.

**Decomposition of the Heterogeneous Effect of Monetary Policy** Figure 8a plots the estimated impulse response function of the difference between the average investment rates of young and old firms to an expansionary monetary policy shock, i.e.,  $\frac{\partial \mathbb{E}(\bar{i}_{Y,t+h} - \bar{i}_{O,t+h})}{\partial \epsilon_t^{MP}}$ . The average investment rate of young firms responds more to a monetary policy shock than that of old firms. This confirms the findings of Cloyne et al. (2023).

Figure 8b decomposes the heterogeneous effect into the contributions arising from the extensive margin ( $\frac{\partial \mathbb{E}(\bar{i}_{Y,t+h}^{ext} - \bar{i}_{O,t+h}^{ext})}{\partial \epsilon_t^{MP}}$ ) and the intensive margin ( $\frac{\partial \mathbb{E}(\bar{i}_{Y,t+h}^{int} - \bar{i}_{O,t+h}^{int})}{\partial \epsilon_t^{MP}}$ ).

**Figure 8:** Decomposition of the Heterogeneous Effect of Monetary Policy



(a) Heterogeneous Effect (Young - Old)

(b) Decomposition: Ext. vs. Int. Margin

*Notes:* Panel (a) of this figure shows the heterogeneous effect of a monetary policy shock on the average investment rate of young firms as opposed to old firms. Panel (b) decomposes this heterogeneous effect into an intensive and an extensive margin contribution, using equation (3). Young (old) firms are less (more) than 15 years old. The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

It shows that the extensive margin explains more than 50% of the heterogeneous average effect on young and old firms.

**Summary of Empirical Evidence** We have documented three main empirical findings. First, monetary policy reshapes the distribution of investment rates. Specifically, an interest rate cut leads to fewer small or zero investment rates and more large investment rates. Second, the change in the distribution is more pronounced among young firms than among old firms. Third, the extensive margin accounts for around 50% of the effect of monetary policy on the *average* investment rate and for more than 50% of the heterogeneous effect on young and old firms.

Appendix C shows that similar but quantitatively less pronounced findings emerge when we compare small and large firms, instead of young and old firms. The second part of the paper presents a heterogeneous-firm model to interpret the empirical findings.

### 3 Model

We build a New Keynesian model with heterogeneous firms subject to fixed and convex capital adjustment costs in the spirit of [Khan and Thomas \(2008\)](#) and [Winberry \(2021\)](#). We introduce firm entry and exit, and consequently, firm life cycles, to study

the cross-section and aggregate implications of firm life cycles for the investment channel of monetary policy.

### 3.1 Investment Block

There exists a continuum of production firms<sup>14</sup> in the economy. Each firm  $j$  produces a quantity  $y_{jt}$  of the intermediate good using the production function

$$y_{jt} = z_{jt} k_{jt}^{\theta} n_{jt}^{\nu} \quad \text{with } \theta, \nu > 0 \text{ and } \theta + \nu < 1 \quad (6)$$

where  $z_{jt}$  is total factor productivity (TFP),  $k_{jt}$  is the capital stock, and  $n_{jt}$  is the labour input. Productivity  $z_{jt}$  is subject to idiosyncratic shocks and follows an AR(1) process in logs

$$\log z_{jt} = \rho_z \log z_{jt-1} + \sigma_z \epsilon_{jt}^z \quad \text{with } \epsilon_{jt}^z \sim \mathcal{N}(0, 1) \quad (7)$$

Labour  $n_{jt}$  can be adjusted frictionlessly in every period. Capital  $k_{jt}$  is accumulated according to

$$k_{jt+1} = (1 - \delta)k_{jt} + i_{jt} \quad (8)$$

where  $i_{jt}$  is investment and  $\delta$  the depreciation rate.

Following [Bachmann et al. \(2013\)](#), we include maintenance investment. That is, a fraction  $\chi$  of the depreciation  $\delta k_{jt}$  that occurs during the production process needs to be replaced immediately. At the end of the period, firms have  $(1 - \delta(1 - \chi))k_{jt}$  units of capital and decide how much to invest voluntarily. To this voluntary investment,  $i_{jt}^v$ , there are capital adjustment costs, which need to be paid if  $i_{jt}^v \neq 0$ .<sup>15</sup> Total adjustment costs consist of a random fixed adjustment cost  $w_t \bar{\zeta}_{jt}$ , where  $\bar{\zeta}_{jt}$  is distributed uniformly between 0 and  $\bar{\zeta}$ , and a convex adjustment cost  $\frac{\phi}{2} \frac{i_{jt}^v{}^2}{k_{jt}}$ :

$$AC(k_{jt}, k_{jt+1}, \bar{\zeta}_{jt}) = w_t \bar{\zeta}_{jt} \mathbb{1}\{k_{jt+1} \neq (1 - \delta(1 - \chi))k_{jt}\} + \frac{\phi}{2} \frac{(k_{jt+1} - (1 - \delta(1 - \chi))k_{jt})^2}{k_{jt}} \quad (9)$$

where  $w_t$  is the real wage. Total investment is the sum of voluntary investment and maintenance investment. The relative price of capital (in terms of the final good) is  $q_t$ .

<sup>14</sup>We normalise the mass of firms to 1. Since entry and exit is exogenous, the mass of firms does not vary in response to aggregate shocks. While our model also features retailers, a final good producer, and a capital good producer, we only refer to intermediate good producers as firms.

<sup>15</sup>Matching the empirical distribution of investment rates requires a rich adjustment cost specification, as discussed in [Cooper and Haltiwanger \(2006\)](#).

**Entry & Exit** Firms face independent and identically distributed (i.i.d.) exit shocks  $\epsilon_{jt}^{exit}$  and are forced to exit the economy at the end of the period with probability  $\pi^{exit}$ . Each period, a fixed mass of newborn firms enters the economy. These entrants are endowed with  $k_0$  units of capital and draw their initial (log) productivity level from the distribution  $\mu^{ent} \sim \mathcal{N}(0, \frac{\sigma_z^2}{1-\rho_z^2})$ , which is the ergodic distribution of (7).

**Timing** Within any period, the timing is as follows. At stage one, idiosyncratic TFP shocks to incumbent firms realise. At stage two, a fixed mass of firms enters the economy. Entrants draw their initial productivity from  $\mu^{ent}$  and are endowed with  $k_0$  units of capital from the household. Henceforth, they are indistinguishable from incumbent firms. At stage three, firms hire labour and production takes place. Firms conduct maintenance investment. At stage four, exit shocks realise and random fixed adjustment costs are drawn. Exiting firms sell their capital stock and leave the economy. Continuing firms decide whether to adjust their capital stock or remain inactive.

**Value Functions** We characterise the firm's optimisation problem recursively. The individual state variables are total factor productivity  $z$  and capital  $k$ . Subscripts for individual variables are henceforth dropped for readability and primes denote next period's values. The beginning-of-period real firm value is

$$V_t(z, k) = \max_n p_t z k^\theta n^\nu - w_t n + \pi^{exit} CV_t^{exit}(z, k) + (1 - \pi^{exit}) \int_0^{\bar{\xi}} CV_t(z, k, \xi) d\xi \quad (10)$$

where  $CV_t^{exit}$  and  $CV_t$  denote the continuation values of exiting and surviving firms, respectively. With probability  $\pi^{exit}$ , a firm is forced to exit after the production stage. Exiting firms have the liquidation value

$$CV_t^{exit}(z, k) = (1 - \delta(1 - \chi))q_t k. \quad (11)$$

as they do not need to pay capital adjustment costs.

The continuation value of a surviving firm is

$$CV_t(z, k, \xi) = \max \{CV_t^a(z, k, \xi), CV_t^n(z, k)\}, \quad (12)$$

which reflects that surviving firms can decide whether to adjust their capital stock ( $CV_t^a$ ) or not ( $CV_t^n$ ). The continuation value of not adjusting is:

$$CV_t^n(z, k) = \mathbb{E}_t [\Lambda_{t+1} V_{t+1}(z', (1 - \delta(1 - \chi))k)] - q_t \chi \delta k, \quad (13)$$

while the continuation value of a firm that adjusts its capital stock is:

$$CV_t^a(z, k, \xi) = \max_{k'} \mathbb{E}_t [\Lambda_{t+1} V_{t+1}(z', k')] - q_t (k' - (1 - \delta)k) - AC(k, k', \xi). \quad (14)$$

**Policy Functions** The labour decision in equation (10) is static and independent of the capital decision

$$n_t^*(z, k) = \left( \frac{p_t v z k^\theta}{w_t} \right)^{\frac{1}{1-v}}. \quad (15)$$

Thus, earnings net of labour costs are

$$\pi_t(z, k) \equiv p_t z k^\theta (n_t^*)^v - w_t n_t^*. \quad (16)$$

The optimal capital decision is computed as follows. First of all, the solution to the maximisation problem in equation (14) is the policy function  $k_t^a(z, k)$ , which is independent of  $\xi$ . This policy function allows us to compute  $CV_t^a(z, k, \xi)$ . Since,  $CV_t^a(z, k, \xi)$  depends on  $\xi$  linearly, we can formulate a cutoff rule for the maximisation problem in equation (12). Firms choose to adjust capital if and only if their fixed adjustment cost draw  $\xi$  is smaller or equal  $\xi_t^T(z, k)$ :

$$k_t^*(z, k, \xi) = \begin{cases} k_t^a(z, k) & \text{if } \xi \leq \xi_t^T(z, k) \\ (1 - \delta(1 - \chi))k & \text{if } \xi > \xi_t^T(z, k) \end{cases} \quad (17)$$

where  $\xi_t^T(z, k) = \frac{CV_t^a(z, k, \xi=0) - CV_t^n(z, k)}{w_t}$ .

We define the hazard rate  $\lambda_t(z, k)$  as:

$$\lambda_t(z, k) = \begin{cases} 0 & \text{if } \xi_t^T(z, k) \leq 0 \\ \frac{\xi_t^T(z, k)}{\bar{\xi}} & \text{if } 0 < \xi_t^T(z, k) \leq \bar{\xi} \\ 1 & \text{if } \bar{\xi} < \xi_t^T(z, k) \end{cases} \quad (18)$$

### 3.2 New Keynesian Block

We separate nominal rigidities from the investment block of the model. A fixed mass of retailers  $i \in [0, 1]$  produces differentiated varieties  $\tilde{y}_{it}$  from the undifferentiated intermediate goods produced by the production firms. There is a one-to-one production technology  $\tilde{y}_{it} = y_{it}$ , where  $y_{it}$  is the amount of the intermediate good that retailer  $i$  purchases. Retailers face Rotemberg quadratic price adjustment costs  $\frac{\varphi}{2} \left( \frac{\tilde{p}_{it}}{\tilde{p}_{it-1}} - 1 \right)^2 Y_t$ , where  $\tilde{p}_{it}$  is the relative price of variety  $i$ .

A representative final good producer aggregates the differentiated varieties optimally into the final good according to

$$Y_t = \left( \int \tilde{y}_{it}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} \quad (19)$$

The resulting demand function for retail good  $\tilde{y}_{it}$  is:

$$\tilde{y}_{it} = \left( \frac{\tilde{p}_{it}}{P_t} \right)^{-\gamma} Y_t \quad (20)$$

where  $P_t = \left( \int_0^1 \tilde{p}_{it}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$  is the price of the final good.

The optimisation problem of a monopolistically competitive retailer  $i$  is:

$$\max_{\{\tilde{p}_{it}\}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \Lambda_t \left\{ (\tilde{p}_{it} - p_t) \tilde{y}_{it} - \frac{\varphi}{2} \left( \frac{\tilde{p}_{it}}{\tilde{p}_{it-1}} - 1 \right)^2 Y_t \right\} \right] \quad (21)$$

subject to the demand curve (20). We log-linearize the optimality condition of the retailer's problem to obtain the familiar New Keynesian Phillips Curve (NKPC):

$$\log(1 + \pi_t) = \frac{\gamma - 1}{\varphi} \log \frac{p_t}{p^*} + \beta \mathbb{E}_t \log(1 + \pi_{t+1}) \quad (22)$$

where  $\pi_t \equiv P_t/P_{t-1} - 1$  is the inflation rate,  $p_t$  is the relative price (in terms of the final good) of the intermediate good, and  $p^* = \frac{\gamma-1}{\gamma}$  is its steady-state value.

### 3.3 Capital Good Producer

There is a representative capital good producer operating in a perfectly competitive market. It transforms units of the final good into new capital subject to external capital adjustment costs:

$$I_t = \left[ \frac{\delta^{1/\kappa}}{1 - 1/\kappa} \left( \frac{I_t^Q}{K_t} \right)^{1-1/\kappa} - \frac{\delta}{\kappa - 1} \right] K_t \quad (23)$$

where  $I_t^Q$  represents the amount of the final good used,  $I_t$  the amount of new capital produced, and  $K_t$  is the total stock of capital in the beginning of period  $t$ . The parameter  $\kappa$  determines the strength of external capital adjustment costs. The static optimization problem is:

$$\max_{I_t} q_t I_t - I_t^Q \quad (24)$$



Optimal behaviour implies that the relative price of capital ( $q_t$ ) has to satisfy the following condition

$$q_t = \left( \frac{I_t^Q / K_t}{\delta} \right)^{1/\kappa} \quad (25)$$

### 3.4 Central Bank

The central bank sets the nominal interest rate  $r_t^m$  according to a Taylor rule

$$\log(1 + r_t^m) = \rho_r \log(1 + r_{t-1}^m) + (1 - \rho_r) \left[ \log \frac{1}{\beta} + \varphi_\pi \log(1 + \pi_t) \right] + \epsilon_t^m \quad (26)$$

where  $\epsilon_t^m$  is a monetary policy shock,  $\rho_r$  is the interest rate smoothing parameter, and  $\varphi_\pi$  is the reaction coefficient to inflation.

### 3.5 Household

There is a representative household, which consumes  $C_t^h$ , supplies labour  $N_t^h$ , and saves or borrows in one-period non-contingent bonds  $B_t^h$ . The household's objective is to maximize expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \log(C_t^h) - \psi N_t^h \right), \quad (27)$$

subject to the flow budget constraint:

$$P_t C_t^h + Q_t^B B_t^h \leq B_{t-1}^h + W_t N_t^h + \Pi_t, \quad (28)$$

where  $Q_t^B$  is the nominal one-period risk-free bond price (one unit of  $B_t$  pays one unit of currency at  $t + 1$ ),  $W_t$  is the nominal wage, and  $\Pi_t$  subsumes additional transfers to and from the household.<sup>16</sup>

Solving the household's optimization problem leads to the following optimality conditions

$$\Lambda_{t+1} \equiv \beta \mathbb{E}_t \left[ \frac{C_t^h}{C_{t+1}^h} \right] \quad (29)$$

$$w_t = \psi C_t^h \quad (30)$$

---

<sup>16</sup> $\Pi_t$  includes dividends from intermediate good producers, retailers, and the final good producer, as well as the initial capital endowment  $k_0$ , which entering firms receive from the household. We follow [Winberry \(2021\)](#) and do not rebate back adjustment costs to the household in a lump-sum manner. Therefore, convex adjustment costs do exhaust the aggregate resource constraint.

where  $\Lambda_{t+1}$  is the household's stochastic discount factor between periods  $t$  and  $t + 1$ , and  $w_t$  is the real wage. Appendix E.1 defines an equilibrium in this economy.

## 4 Quantitative Results

We use the model to inspect the investment channel of monetary policy. Section 4.1 calibrates the model. Section 4.2 analyses the effects of a monetary policy shock, mirroring the empirical analysis. Section 4.3 studies the aggregate implications of firm heterogeneity in the model. Section 4.4 discusses the fit between model and data.

### 4.1 Calibration

We calibrate the model to the US economy. Wherever possible, we rely on data sources that are representative of the entire economy. We begin by fixing a subset of parameters to conventional values. These parameters are summarised in Table A.1. Given these fixed parameters, we fit the remaining parameters to match the moments listed in Table 2. The fitted parameters are listed in Table 1.

Since a model period corresponds to a quarter, the discount factor is set to  $\beta = 0.99$ . The labour disutility parameter is set to  $\psi = 0.55$ .<sup>17</sup> Capital and labour coefficients are set to standard values, that is,  $\theta = 0.21$  and  $\nu = 0.64$  (Ottonello and Winberry, 2020). The depreciation rate  $\delta = 1.93\%$  generates an annual aggregate investment rate of 7.7% as reported in Zwick and Mahon (2017). We target the standard deviation of idiosyncratic TFP shocks  $\sigma_z$ , but fix their persistence  $\rho_z$  due to the identification problem discussed in Clementi and Palazzo (2015). We set  $\rho_z$  to 0.95 (Khan and Thomas, 2008; Bloom et al., 2018). The exit probability  $\pi^{exit}$  is set to 1.625% as in Koby and Wolf (2020).<sup>18</sup> We choose standard values for the parameters of the New Keynesian block, i.e.  $\varphi = 90$  and  $\gamma = 10$  (Ottonello and Winberry, 2020). The coefficient on inflation in the Taylor rule  $\varphi_\pi$  is set to 1.5 and the interest rate smoothing parameter  $\rho_r$  is set to 0.75. External capital adjustment costs  $\kappa$  are set to 11 to roughly match the peak effect of a monetary policy shock on investment relative to the peak effect on output documented empirically (Figure D.1).

The five parameters listed in Table 1 are chosen to match five targeted moments listed in Table 2. Even though all parameters are calibrated jointly, we briefly explain which moments are particularly informative about the respective parameters. First, we target the standard deviation of investment rates, because it is informative about

<sup>17</sup>This value follows from normalising the steady-state real wage  $w$  to 1.

<sup>18</sup>This exit probability brings the age distribution as close to the data as possible without using age-specific exit probabilities.

**Table 1: Fitted Parameters**

| Parameter   | Description                          | Value |
|-------------|--------------------------------------|-------|
| $\sigma_z$  | Volatility of TFP Shock              | 0.07  |
| $k_0$       | Initial Capital of Entrants          | 2.73  |
| $\bar{\xi}$ | Upper Bound on Fixed Adjustment Cost | 0.96  |
| $\phi$      | Convex Adjustment Cost               | 2.20  |
| $\chi$      | Maintenance Investment Parameter     | 0.30  |

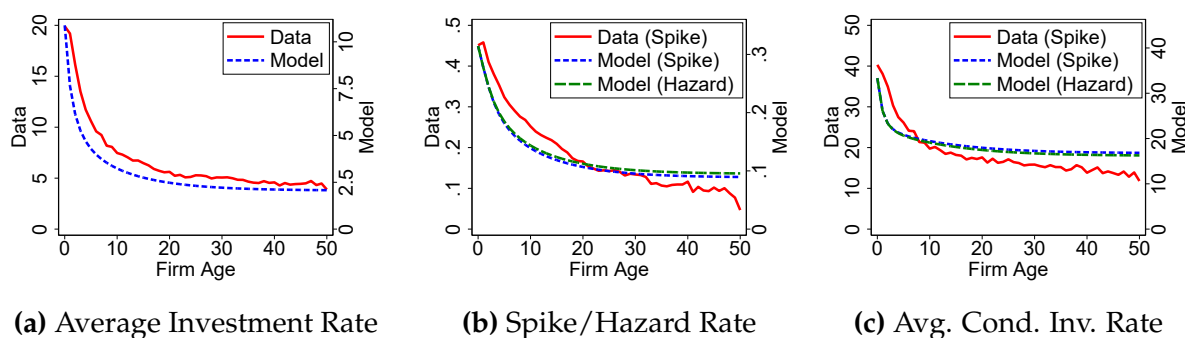
**Table 2: Empirical & Simulated Moments**

| Moment                                 | Data | Model |
|--|------|-------|
| Standard Deviation of Investment Rates | 0.20 | 0.18  |
| Average Investment Rate                | 0.12 | 0.13  |
| Autocorrelation of Investment Rates    | 0.38 | 0.37  |
| Relative Size of Entrants              | 0.29 | 0.29  |
| Relative Spike Rate of Old Firms       | 0.40 | 0.38  |

*Notes:* Data moments related to investment rates are taken from [Zwick and Mahon \(2017\)](#) (Appendix, Table B.1, Unbalanced Sample). The relative spike rate of old firms is computed from Compustat data. Corresponding model moments are computed from a simulation of a large panel of firms. The relative size of entrants is taken from Business Dynamics Statistics (BDS). In the model, this moment can be computed from the steady-state distribution.

the volatility of idiosyncratic TFP shocks. Second, we target the average investment rate as it is informative about both adjustment cost parameters. Increasing either adjustment cost dampens investment rates in particular of young firms and therefore the average investment rate. Third, we target the autocorrelation of investment rates, because it is informative about the relative importance of fixed and convex adjustment costs. Convex adjustment costs generate a positive autocorrelation, whereas fixed adjustment costs generate a negative or zero autocorrelation. For these three moments, we use the statistics reported in [Zwick and Mahon \(2017\)](#). Fourth, we target the relative size of entrants, which is informative about the initial capital of entrants. This moment is computed from Business Dynamics Statistics (BDS) data. Fifth, we target the spike rate of old firms relative to the spike rate of young firms, which is informative about the maintenance investment parameter. The more depreciation is undone by maintenance investment, the less frequently do old firms need to make an extensive margin investment. Thus, a higher maintenance parameter leads to a lower spike rate among old firms. This moment needs to be computed from Compustat data, since it is the only data source which includes both investment rates and firm age.

**Figure 9: Life Cycle Profiles**



*Notes:* Investment, hazard, and spike rates refer to a quarter. The spike rate is defined as the fraction of firms choosing an investment rate larger than 10%. The hazard rate, observable only in the model, is defined as the fraction of firms choosing to pay the fixed adjustment cost and adjust their capital stock. Panel (c) plots the average investment rate among all firms that choose an investment rate larger than 10% or among all firms paying the fixed adjustment costs.

#### 4.1.1 Untargeted Moments

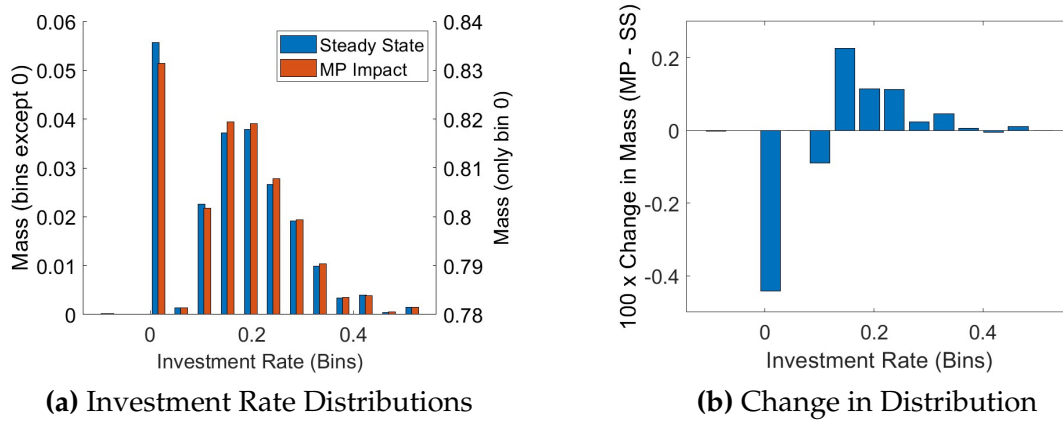
At the calibrated parameters, the simulated moments match the targeted empirical moments well (Table 2). Before moving to the main analysis, we show that the model is also capable of reproducing well-known facts regarding (i) firm life cycles, (ii) the aggregate effects of monetary policy shocks, and (iii) the interest-rate-elasticity of aggregate investment. The model being able to match these untargeted moments serves as an external validation for the calibration of the model.

**Firm Life Cycle Profiles** Figure 9 shows that the model matches the life cycle profiles of firm investment behaviour very well. Panel (a) shows that in the data as in the model, the average investment rate is highest for newborn firms and falls monotonically in age. Panels (b) and (c) decompose the pattern of the average investment rate into the spike rate (hazard rate) and the average investment rate conditional on an investment spike (on an adjustment of the capital stock). Evidently, the observation that young firms have higher average investment rates is driven in part by higher spike rates (hazard rates) and in part by higher conditional investment rates. In the model, we use the hazard rate, whereas, in the data, we rely on the spike rate as a proxy for the hazard rate. Panels (b) and (c) show that the difference between the two is quantitatively negligible.<sup>19</sup> While the life cycle profiles in the data and the model align well, there are some differences in levels, which are discussed in Section 4.4.

It is worth emphasising that all three investment frictions are necessary to generate

<sup>19</sup>Effectively, this means that in the model there are very few firms which choose to pay the fixed adjustment cost and then choose an investment rate that is smaller than 10%.

**Figure 10:** Effect of Monetary Policy on the Distribution of Investment Rates



*Notes:* Panel (a) plots the distribution of investment rates in steady state (blue bars) and after an expansionary monetary policy shock (red bars). To improve readability, the shock is scaled to reduce the nominal interest rate by 100 basis points. Panel (b) plots the difference between the two distributions shown in Panel (a).

these life cycle profiles. First, fixed adjustment costs generate lumpy investment behaviour, so hazard rates below one, as shown in Panel (b). Second, convex adjustment costs ensure that young firms in the model choose plausible conditional investment rates and do not immediately jump to their optimal size, as shown in Panel (c). Third, maintenance investment makes hazard rates decrease with age, as shown in Panel (b).

**The Aggregate Effects of Monetary Policy Shocks** We study the effects of an unexpected expansionary monetary policy shock followed by a perfect foresight transition back to steady state.<sup>20</sup> Figure A.6 plots the impulse response functions of aggregates and prices, which confirm that our model produces the typical New Keynesian effects to a monetary policy shock.

**Interest-Rate-Elasticity of Aggregate Investment** Koby and Wolf (2020) have shown that the aggregate relevance of firm heterogeneity hinges on a low elasticity of aggregate investment with respect to the interest rate. Based on quasi-experimental evidence, their preferred estimate for this elasticity at annual frequency is -5, whereas, in models without aggregate relevance of firm heterogeneity it can be as high as -500 (Khan and Thomas, 2008). In our baseline calibration, this elasticity is -8.2, a value

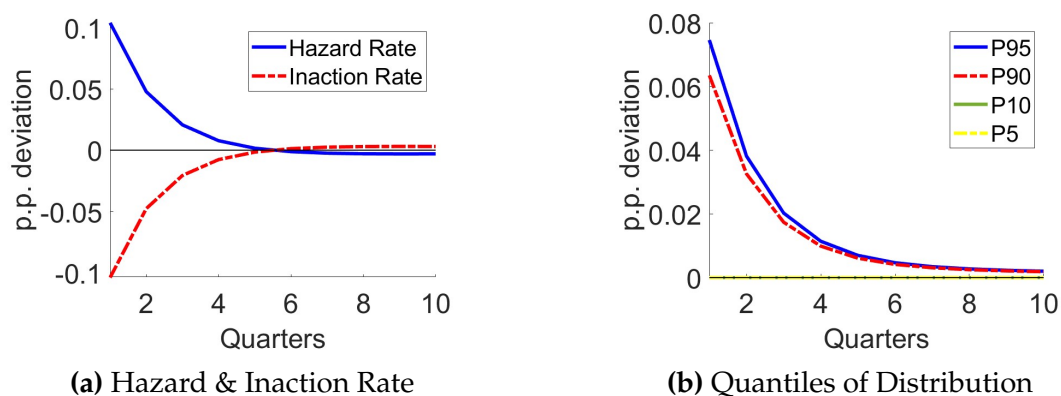
<sup>20</sup>This approach to constructing impulse response functions to aggregate shocks follows Boppart et al. (2018). The size of the monetary shock is chosen to roughly match the peak effects on output and investment seen in the data. This implies that the nominal interest rate falls by around 25 basis points on impact.

close to [Koby and Wolf \(2020\)](#) which implies that firm heterogeneity matters for aggregate dynamics, as studied in more detail in Section 4.3.

## 4.2 Monetary Policy and the Distribution of Investment Rates

Turning to the main analysis of this paper, Figure 10 plots the effect of a monetary policy shock on the distribution of investment rates. Specifically, it plots the distribution of investment rates in steady state (blue bars) and in the period when an expansionary monetary policy shock has hit the economy (red bars). It is apparent that monetary policy affects some firms' extensive margin investment decision and therefore the distribution of investment rates: after an interest rate cut, there are fewer inactive firms and more firms choosing to make an investment. This observation corresponds to **Fact 1** documented in Section 2.3. Moreover, Figure 11 shows that the hazard rate rises, the inaction rate falls, and the upper quantiles of the investment rate distribution respond substantially more than the lower quantiles, matching the empirical evidence (see Figures 2 and 4).

**Figure 11:** Effect of Monetary Policy on Hazard Rate, Inaction Rate, Quantiles

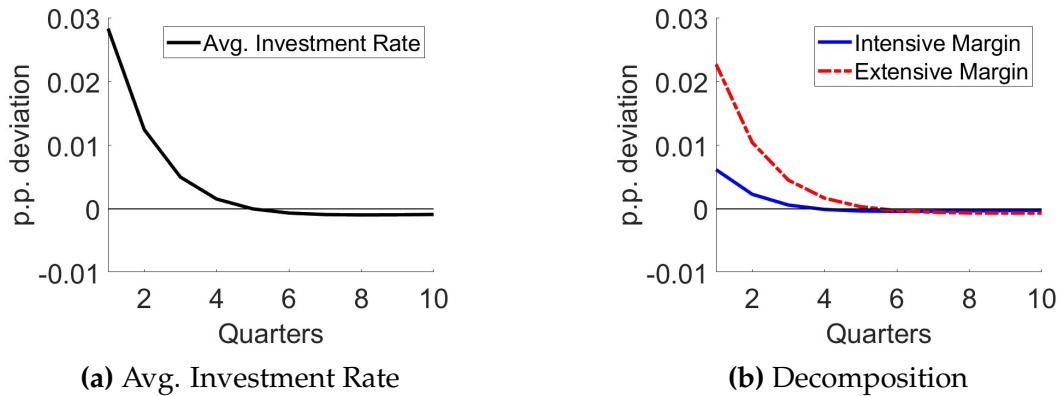


*Notes:* Panel (a) of this figure plots the effect of a monetary policy shock on the hazard and inaction rate of all firms. Panel (b) plots the IRFs of several quantiles of the investment rate distribution.

Monetary policy affects the average investment rate not only via the extensive margin but also via the intensive margin. To assess the relative importance of both margins, we decompose the effect on the average investment rate into contributions of the extensive and intensive margin, similar to the empirical exercise presented in Figure 7.<sup>21</sup> Panel (a) of Figure 12 shows that the expansionary monetary policy shock

<sup>21</sup>This decomposition is computed by holding either hazard rates at steady-state levels (intensive margin contribution) or investment rates conditional on investing at steady-state levels (extensive margin contribution), see Equation (3).

**Figure 12: Effects of Monetary Policy: Extensive & Intensive Margin**



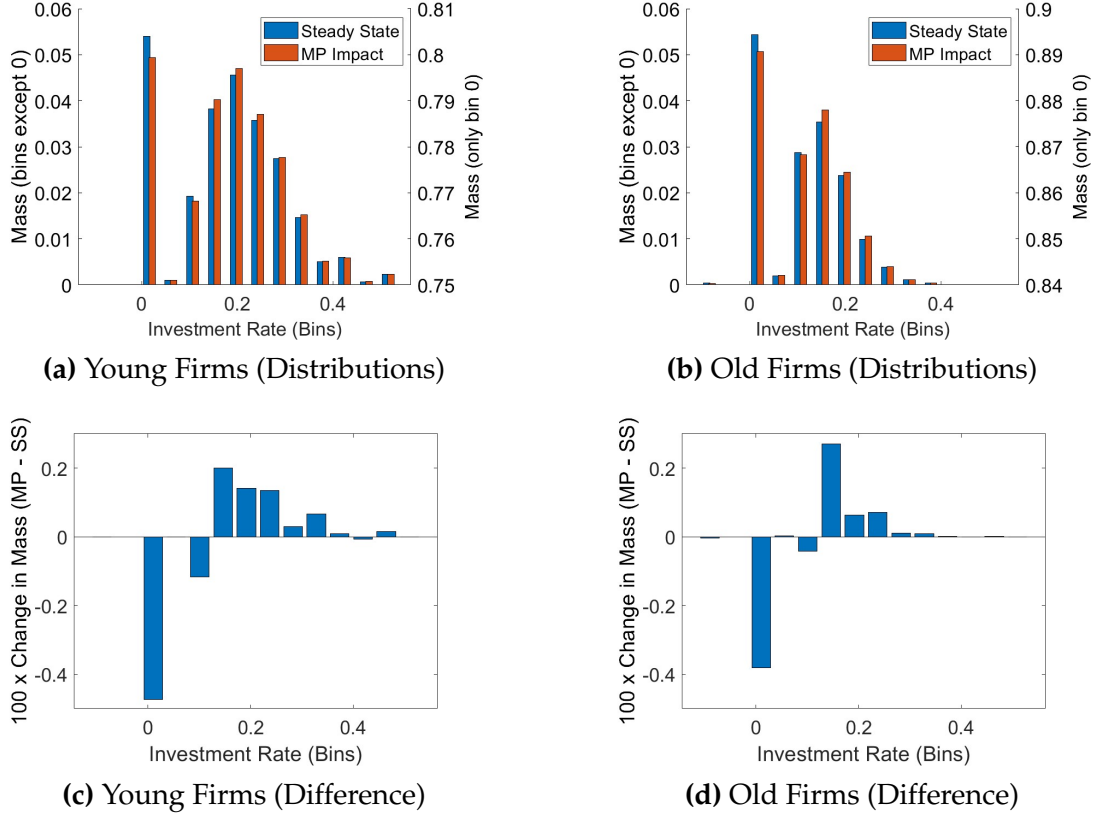
*Notes:* Panel (a) of this figure plots the effect of a monetary policy shock on the average investment rate of all firms. Panel (b) decomposes the IRF in panel (a) into an extensive margin contribution and an intensive margin contribution, by holding conditional investment rates (extensive margin only) or hazard rates (intensive margin only) at their steady-state levels.

increases the average investment rate, while Panel (b) plots the decomposition. The model attributes a significant portion of the change in the average investment rate to the extensive margin, as in the data.

**Heterogeneous Effects: Young vs. Old Firms** In addition, the model reproduces the empirical finding that the effect of monetary policy on the distribution of investment rates is heterogeneous across age groups, as shown in Figure 13. This corresponds to **Fact 2**. Panels (a) and (b) plot the distribution of investment rates before and after an expansionary monetary policy shock of young and old firms, respectively. The bottom panels plot the changes in the distribution, highlighting that after an interest rate cut, there are more young firms than old firms switching from being inactive to making a large investment. That is, the investment channel of monetary policy along the extensive margin is more pronounced among young firms.

Due to the heterogeneous effect along the extensive margin, monetary policy affects *average* investment rates differently across age groups. Panel (a) of Figure 14 shows that after an expansionary shock young firms increase their investment rates on average more strongly than old firms. Panel (b) decomposes this heterogeneous effect into extensive and intensive margin contributions, similar to the empirical exercise shown in Figure 8b, and demonstrates that the total difference is driven by the extensive margin. Panel (c) shows that one reason for this heterogeneous effect along the extensive margin is the larger increase in the hazard rate among young firms, shown in the data in Figure 6. However, this *heterogeneous hazard rate increase* is not the only

**Figure 13:** Effect of Monetary Policy on the Distribution of Inv. Rates (by Age Group)



*Notes:* Panels (a) and (b) of this figure plot the distribution of investment rates of young (old) firms in steady state (blue bars) and after a monetary policy shock (red bars). To improve readability, the shock is scaled to reduce the nominal interest rate by 100 basis points. Panels (c) and (d) plot the difference of the two distributions for young (old) firms.

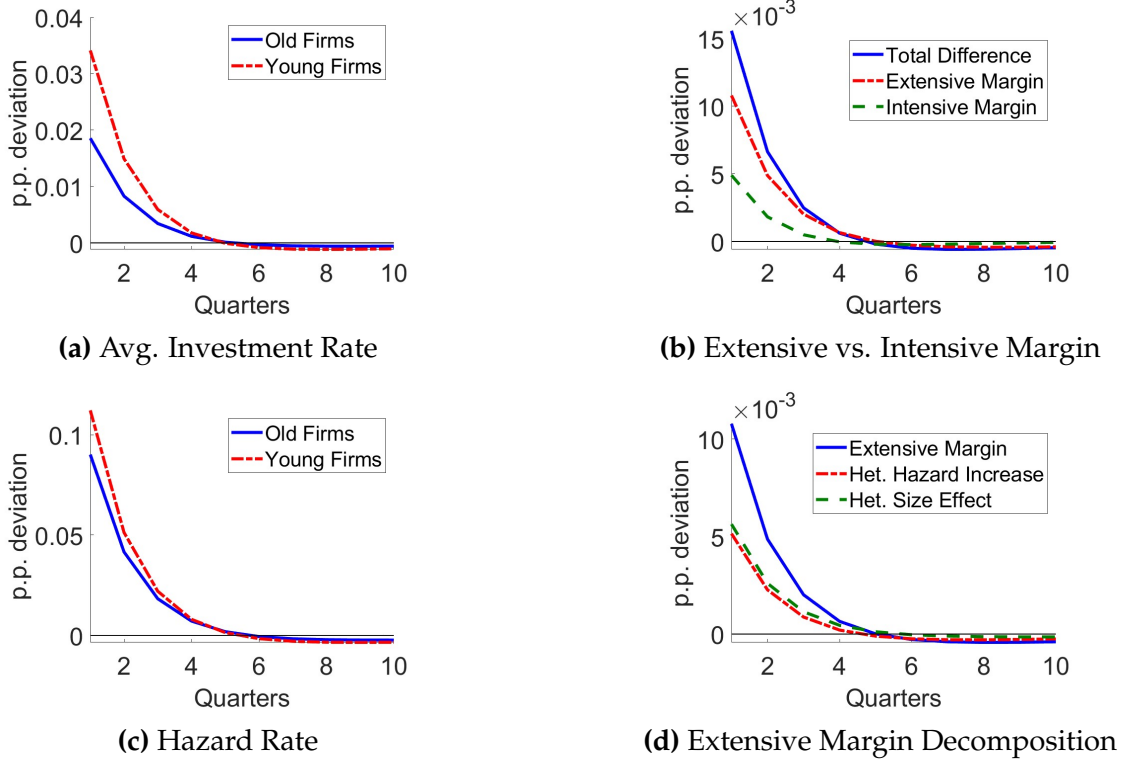
reason, as the additional young adjusters on average also choose a higher investment rate than the additional old adjusters (also visible in Panels (c) and (d) of Figure 13). We refer to this as the *heterogeneous size effect*. Panel (d) shows that both effects are quantitatively important.

**Intuition** We now build intuition for the extensive margin investment channel, while in Appendix B, we derive the *heterogeneous size effect* and the *heterogeneous hazard rate increase* analytically in a stylized two-period model. Drawing on the decomposition of the group-specific average investment rate already used in Section 2.5, the average investment rate ( $\bar{i}$ ) among firms of group  $g$  is

$$\bar{i}_g = \psi_g i_g^* + (1 - \psi_g) i^m \quad (31)$$



**Figure 14: Heterogeneous Effect of Monetary Policy (by Age Group)**



*Notes:* Panel (a) of this figure plots the effect of a monetary policy shock on the average investment rates of young and old firms. Panel (b) decomposes the differences of the two IRFs in panel (a) into an extensive margin contribution and an intensive margin contribution. Panel (c) plots the effect of a monetary policy shock on the hazard rates of young and old firms. Panel (d) further decomposes the IRF of the extensive margin contribution in panel (b) into the heterogeneous hazard rate increase and the heterogeneous size effect.

where  $\psi_g$  is the hazard rate,  $i_g^*$  is the average investment rate among firms paying the fixed cost, and  $i^m$  is the time-invariant (maintenance) investment rate of firms not paying the fixed cost.

The interest rate sensitivity of the average investment rate is:

$$\frac{\partial \bar{i}_g}{\partial r} = \underbrace{\frac{\partial \psi_g}{\partial r} (i_g^* - i_g^m)}_{\text{Extensive Margin}} + \underbrace{\psi_g \frac{i_g^*}{\partial r}}_{\text{Intensive Margin}} \quad (32)$$

A decrease in the interest rate leads to a higher hazard rate ( $\frac{\partial \psi_g}{\partial r} < 0$ ) because it increases the discounted benefit of investing while leaving the cost of investing (in partial equilibrium) unchanged, leading to more firms paying the fixed cost. This is what we label the extensive margin investment channel.

Comparing the increase in the average investment rate due to the extensive margin

among young (Y) and old (O) firms, we uncover the two effects plotted in Panel (d) of Figure 14:

$$\begin{aligned}
\text{HetExt}_{Y-O} &= \underbrace{\frac{\partial \psi_Y}{\partial r} (i_Y^* - i^m)}_{\text{Young Firms}} - \underbrace{\frac{\partial \psi_O}{\partial r} (i_O^* - i^m)}_{\text{Old Firms}} \\
&= \underbrace{\frac{\partial \psi_O}{\partial r} (i_Y^* - i_O^*)}_{\text{Heterogeneous Size Effect}} + \underbrace{\left( \frac{\partial \psi_Y}{\partial r} - \frac{\partial \psi_O}{\partial r} \right) (i_Y^* - i^m)}_{\text{Heterogeneous Hazard Rate Increase}} \quad (33)
\end{aligned}$$

On the one hand, there is the *heterogeneous size effect*. Among the *new* adjusters, young firms choose higher investment rates conditional on adjusting than old firms ( $i_Y^* - i_O^* > 0$ ). Panel (c) of Figure 9 has shown that in the data and in the model, young firms have on average higher conditional investment rates. Therefore, there would be a heterogeneous effect on average investment rates even if an interest rate cut had the same effect on hazard rates of young and old firms.

On the other hand, there is a *heterogeneous hazard rate increase* as an interest rate cut raises the hazard rate of young firms by more than the hazard rate of old firms. In general, hazard rates rise because the discounted benefit of investing rises, while the cost remains unchanged.<sup>22</sup> This increase in the discounted benefit of investing is larger for young firms. The reason is that young firms have a higher marginal product of capital, which reflects that young firms are farther away from their “optimal size” as they are on average smaller and the model features decreasing returns to scale. Therefore, young firms are more inclined to make an investment, and monetary policy can more easily induce them to do so. This mechanism echoes [Baley and Blanco \(2021\)](#), who have shown that capital misallocation is a key statistic for the investment dynamics following an aggregate shock. According to the *heterogeneous hazard rate increase*, young firms are more responsive to interest rate changes than old firms because misallocation—measured here by the average distance to optimal size—is larger among young firms than among old firms.

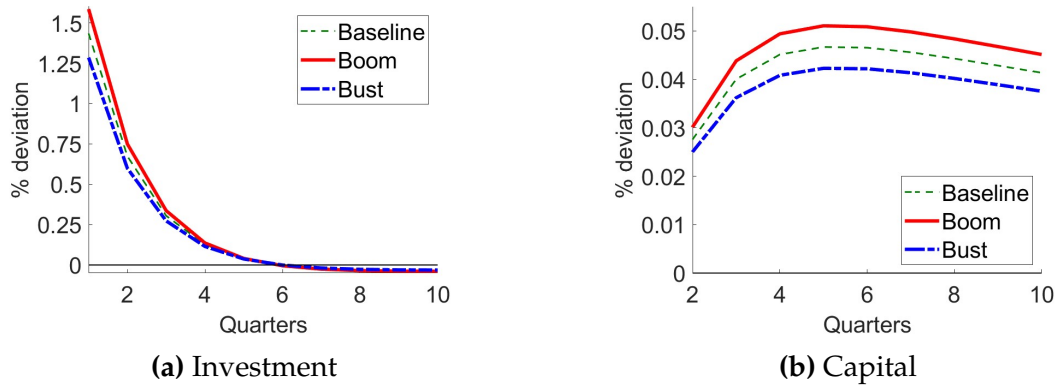
### 4.3 Aggregate Implications

The features of firm-level investment behaviour—lumpiness and life cycle dynamics—that we emphasise to be important to match the empirical facts have aggregate implications. Lumpy investment behaviour implies that monetary policy is particularly

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<sup>22</sup>The discounted benefit of investing is  $\frac{1}{1+r} \left( V(k_0 \times (1 + i^*)) - V(k_0 \times (1 - \delta(1 - \chi))) \right)$ . Due to general equilibrium effects, the cost of investing is affected by interest rate changes as well, but there is no direct (partial equilibrium) effect.

**Figure 15: State-Dependent Effects of Monetary Policy**



*Notes:* This figure plots the effects of a monetary policy shock on aggregate investment and capital. For the boom (bust) impulse response functions, the monetary policy shock is combined with a TFP shock that increases (decreases) TFP on impact by 5%.

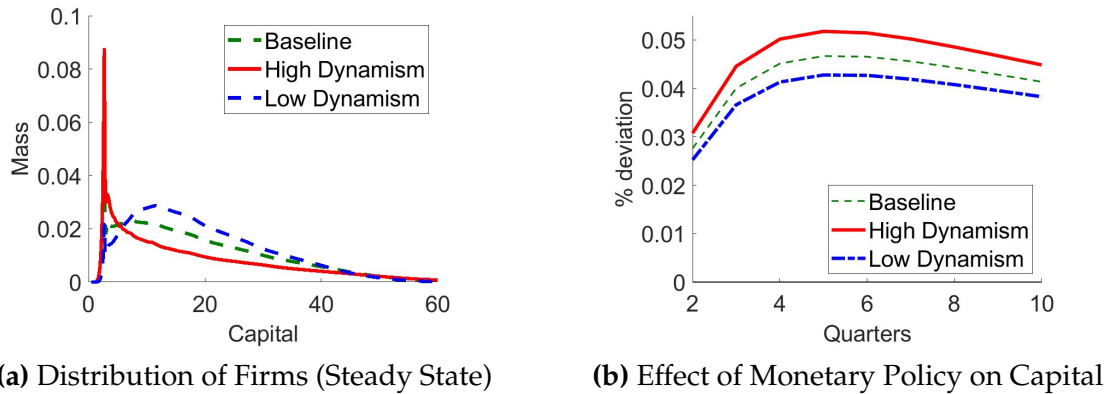
effective in stimulating investment whenever there are many firms that are “close to paying the fixed cost”, as they can be induced by monetary policy to make a meaningful investment. As also age matters for investment behaviour, our model implies that the effectiveness of monetary policy varies over the business cycle as well as due to long-run trends in the age distribution of firms.

First, monetary policy is more effective in a boom than in a bust. In a boom, productivity and the return on capital are high and therefore many firms are “close to paying the fixed cost” and can be induced to invest. Figure 15 shows that the increase in investment following the same monetary policy shock is about 23% more effective in a large boom than in a deep recession. This aligns well with the findings of [Koby and Wolf \(2020\)](#).

Second, firm aging affects the investment channel of monetary policy. Specifically, monetary policy is more effective when business dynamism is high (high entry and exit rates). This is because when business dynamism is high, the share of young firms in the economy is higher, and young firms are more easily induced to pay the fixed cost and invest. In our baseline calibration, the share of entrants (the number of firms of age 0 relative to the number of all firms) is around 6.5% and thus close to the value observed in the US over the past decade. To quantify the relevance of the decline in business dynamism, documented by [Haltiwanger et al. \(2012\)](#) among others, for monetary policy, we compute a counterfactual “high-dynamism” calibration, which features a twice as high share of entrants, as observed in 1984 (13%).<sup>23</sup> We also compute

<sup>23</sup>To do so, we double both entry and exit rates while holding all other model parameters fixed. This ensures that the mass of firms remains 1, but the distribution of firms features more young firms.

**Figure 16:** Firm Aging Affects the Investment Channel of Monetary Policy



Notes: Panel (a) of this figure shows the steady-state distributions of firms over capital. The underlying capital grid is log-spaced. Panel (b) shows the impulse response function of aggregate capital to an expansionary monetary policy shock. The three calibrations only differ with respect to the entry/exit rate which is 6.5% (baseline), 13% (high dynamism), and 3.375% (low dynamism).

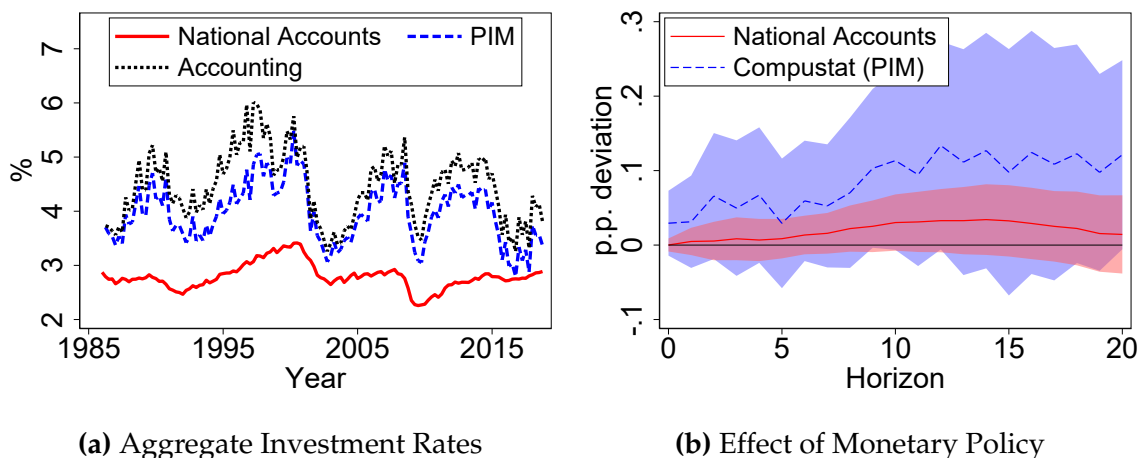
a “low-dynamism” calibration, which features a 50% lower share of entrants (3.375%). In the steady state of the high-dynamism (low-dynamism) calibration, there are relatively more (fewer) young and therefore small firms, as Panel (a) of Figure 16 shows. Since young firms are more responsive to interest rate changes, the impact effect of a monetary policy shock on aggregate investment, which is 1.42% in the baseline calibration, is with 1.59% around 12% larger in the high-dynamism calibration and with 1.30% around 8.5% smaller in the low-dynamism calibration. Panel (b) of Figure 16 shows that as these differential effects persist over the first year, the total capital stock has after four quarters grown by 11% more (8.5% less) in the high-dynamism (low-dynamism) calibration compared to the baseline. Hence, according to our model, the well-documented decline in business dynamism has made monetary policy less effective in stimulating investment.

#### 4.4 Discussion: Model vs. Data

Our model replicates the two empirical findings qualitatively very well, while there are some quantitative differences worth discussing. First, in the data (Figure A.2, Panel a) the average investment rate of young firms is about 3.5 times as responsive to a monetary shock as the average investment rate of old firms. In the model (Figure 14, Panel a), the average investment rate of young firms is only about 2 times as responsive. This shows that the calibrated lumpy investment model can explain a sizable share, but not all of the heterogeneous effect of monetary policy on young and old firms, suggesting the presence of other quantitatively relevant mechanisms, such as

financial acceleration.

**Figure 17:** Aggregate Investment Rates (NIPA & Compustat Data)



Notes: Panel (a) plots three quarterly aggregate investment rates. The first one is computed from national accounts data, following the procedure described in [Bachmann et al. \(2013\)](#). The other two are constructed from Compustat firms, reflecting two alternative ways of constructing capital. The first one uses investment and capital as computed with the perpetual inventory method (“PIM”). The second one uses investment and capital as reported in Compustat (“Accounting”). Both Compustat investment rates are seasonally adjusted using quarterly dummy variables to deal with the observation that reported investment rates are typically higher in the fourth quarter ([Xu and Zwick, 2021](#)). Panel (b) plots the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

Second, while the model matches well the effect of monetary policy on aggregate investment (Figures [D.1](#) and [A.6](#)), the effect on the average investment rate (Panel (a) of Figures [7](#) and [12](#)) is substantially smaller. This primarily reflects an important discrepancy between investment data from Compustat, used to estimate this effect in the data, and investment data from national accounts, which is used to calibrate the model.<sup>24</sup> In particular, the aggregate investment rate in Compustat is substantially higher and more volatile than the aggregate investment rate constructed from national accounts data, as shown in Panel (a) of Figure [17](#).<sup>25</sup> In light of this, it is not surprising to find different estimates regarding the effect of monetary policy. Figure [17](#) plots the

<sup>24</sup>Following other studies in the literature, we use Compustat data because it offers quarterly firm-level data including information on investment rates and firm age. However, Compustat firms, being public firms, are by no means a random or representative sample of the universe of firms in the economy, giving rise to this discrepancy.

<sup>25</sup>Aggregate investment rates from Compustat having a higher level at least partly reflects the capital measurement issues described in Appendix [D.3](#). The PIM addresses these issues to some extent, but the level of the investment rate remains substantially above the national-accounts investment rate. Despite the differences in the level and volatility, the investment rates are highly correlated. The aggregate investment rate from national accounts has a correlation of  $\rho = 0.6$  with the “PIM” investment rate

impulse response functions of aggregate investment rates from national accounts and from Compustat (PIM) data. While the trajectory is very similar, the magnitude differs substantially. The peak effects are about 0.03 percentage points (national accounts) and 0.13 percentage points (Compustat PIM), respectively. Since our model is calibrated to national accounts data, it quantitatively matches the former number, not the latter one.

Thanks to calibrating the model to the national accounts data, the aggregation implications presented in Section 4.3 are relevant for the U.S. economy as a whole rather than a subset of the economy (publicly listed firms).

## 5 Conclusion

In this paper, we highlight two features of firm-level investment behaviour that are important to understand the investment channel of monetary policy. First, investment is lumpy at the firm level and therefore, there is a quantitatively relevant investment channel along the extensive margin. That is, an interest rate cut induces some firms to switch from not investing to making a meaningful investment. Second, life cycle investment dynamics are important to understand the heterogeneous effects of monetary policy on firms of different age groups. Young firms tend to grow whereas many old firms have reached their optimal size. Therefore, young firms invest larger amounts and more frequently and are also more easily induced by monetary policy to invest.

We present three pieces of evidence in line with these mechanisms. First, monetary policy affects the shape of the distribution of investment rates. Specifically, an interest rate cut leads to fewer small and zero investment rates and more large investment rates. Second, this change in the distribution is more pronounced among young firms than among old firms. Third, a decomposition exercise indicates that the extensive margin accounts for around 50% of the effect of monetary policy on the average investment rate and for more than 50% of the heterogeneous effect on firms of different age groups.

We build a heterogeneous-firm model that combines fixed adjustment costs, firm life cycle dynamics, and nominal rigidities to rationalise these novel empirical findings. In the model, fixed adjustment costs give rise to lumpy investment behaviour and an investment channel of monetary policy along the extensive margin. Young firms can more easily be induced to make an investment because they are farther away

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and of  $\rho = 0.54$  with the “Accounting” investment rate. Both Compustat investment rates are highly correlated ( $\rho = 0.95$ ).

from their optimal size than old firms. The model allows us not only to rationalise the empirical evidence but also to investigate the aggregate implications of the key features of the model—lumpy investment and firm life cycle dynamics. We show that the effectiveness of monetary policy varies over the business cycle and also due to long-run trends. In particular, the secular decline in business dynamism has weakened the investment channel.

Our work highlights an important avenue for future research, guided by three questions: Why do young firms grow slowly? How are the relevant frictions affected by economic policy? In turn, how are the effects of economic policies determined by these frictions? A long literature has emphasised financial frictions as the key constraint for young firms (Gertler and Gilchrist, 1994; Kochen, 2023). We show that fixed capital adjustment costs—generating lumpy investment behaviour—are another key constraint for young firms and determine the effectiveness of monetary policy. Yet, there are further non-financial factors that constrain particularly young firms, such as uncertainty about productivity and demand (Jovanovic, 1982; Chen et al., 2023). Gaining a better and more complete understanding of why young firms grow slowly and how the relevant frictions matter for economic policy is crucial to guiding the design of effective policy interventions in the future.

## References

- Asriyan, V., L. Laeven, A. Martin, A. Van der Ghote, and V. Vanasco (2022). Falling interest rates and credit reallocation: Lessons from general equilibrium. *CEPR Discussion Paper 16720*.
- Bachmann, R. and C. Bayer (2014). Investment Dispersion and the Business Cycle. *American Economic Review* 104(4), 1392–1416.
- Bachmann, R., R. J. Caballero, and E. M. R. A. Engel (2013). Aggregate Implications of Lumpy Investment: New Evidence and a DSGE Model. *American Economic Journal: Macroeconomics* 5(4), 29–67.
- Baley, I. and A. Blanco (2021). Aggregate dynamics in lumpy economies. *Econometrica* 89(3), 1235–1264.
- Belo, F., X. Lin, and S. Bazdresch (2014). Labor hiring, investment, and stock return predictability in the cross section. *Journal of Political Economy* 122(1), 129–177.
- Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2018). Really Uncertain Business Cycles. *Econometrica* 86(3), 1031–1065.

- Boppart, T., P. Krusell, and K. Mitman (2018). Exploiting MIT shocks in heterogeneous-agent economies: the impulse response as a numerical derivative. *Journal of Economic Dynamics and Control* 89, 68–92.
- Caballero, R. J. and E. M. Engel (1999). Explaining investment dynamics in US manufacturing: a generalized (S, s) approach. *Econometrica* 67(4), 783–826.
- Caballero, R. J., E. M. Engel, and J. C. Haltiwanger (1995). Plant-level adjustment and aggregate investment dynamics. *Brookings Papers on Economic Activity*, 1–54.
- Chen, C., T. Senga, C. Sun, and H. Zhang (2023). Uncertainty, imperfect information, and expectation formation over the firm’s life cycle. *Journal of Monetary Economics*.
- Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy* 113(1).
- Clementi, G. L. and B. Palazzo (2015). On The Calibration of Competitive Industry Dynamics Models . *Mimeo, New York University*.
- Cloyne, J., C. Ferreira, M. Froemel, and P. Surico (2023). Monetary policy, corporate finance, and investment. *Journal of the European Economic Association*.
- Cooper, R., J. Haltiwanger, and L. Power (1999). Machine Replacement and the Business Cycle: Lumps and Bumps. *American Economic Review* 89(4), 921–946.
- Cooper, R. W. and J. Haltiwanger (2006). On the Nature of Capital Adjustment Costs. *Review of Economic Studies* 73(3), 611–633.
- Crouzet, N. and N. R. Mehrotra (2020). Small and large firms over the business cycle. *American Economic Review* 110(11), 3549–3601.
- Fang, M. (2023). Lumpy Investment, Fluctuations in Volatility and Monetary Policy. *Mimeo, University of Florida*.
- Farre-Mensa, J. and A. Ljungqvist (2016). Do measures of financial constraints measure financial constraints? *Review of Financial Studies* 29(2), 271–308.
- Fee, C. E., C. J. Hadlock, and J. R. Pierce (2009). Investment, Financing Constraints, and Internal Capital Markets: Evidence from the Advertising Expenditures of Multinational Firms. *The Review of Financial Studies* 22(6), 2361–2392.
- Gertler, M. and S. Gilchrist (1994). Monetary policy, business cycles, and the behavior of small manufacturing firms. *Quarterly Journal of Economics* 109(2).



- Gertler, M. and P. Karadi (2015). Monetary Policy Surprises, Credit Costs and Economic Activity. *American Economic Journal: Macroeconomics* 7(1), 44–76.
- González, B., G. Nuño, D. Thaler, and S. Albrizio (2022). Firm heterogeneity, capital misallocation and optimal monetary policy. *Banco de Espana Working Paper* 2145.
- Gourio, F. and A. K. Kashyap (2007). Investment spikes: New facts and a general equilibrium exploration. *Journal of Monetary Economics* 54, 1–22.
- Hadlock, C. J. and J. R. Pierce (2010). New Evidence on Measuring Financial Constraints: Moving Beyond the KZ Index. *The Review of Financial Studies* 23(5), 1909–1940.
- Haltiwanger, J., R. Jarmin, and J. Miranda (2012). Where have all the young firms gone? *Business Dynamics Statistics Briefing* 6.
- House, C. L. (2014). Fixed costs and long-lived investments. *Journal of Monetary Economics* 68(1), 86–100.
- Jeenas, P. (2023). Firm Balance Sheet Liquidity, Monetary Policy Shocks, and Investment Dynamics. *Mimeo, Universitat Pompeu Fabra*.
- Jovanovic, B. (1982). Selection and the evolution of industry. *Econometrica*.
- Jungherr, J., M. Meier, T. Reinelt, and I. Schott (2022). Corporate debt maturity matters for monetary policy. *Mimeo, University of Bonn*.
- Khan, A. and J. Thomas (2008). Idiosyncratic shocks and the role of nonconvexities. *Econometrica* 76(2), 395–436.
- Khan, A. and J. K. Thomas (2003). Nonconvex factor adjustments in equilibrium business cycle models: do nonlinearities matter? *Journal of Monetary Economics* 50(2), 331–360.
- Koby, Y. and C. Wolf (2020). Aggregation in heterogeneous-firm models: Theory and measurement. *Mimeo, Princeton University*.
- Kochen, F. (2023). Finance over the life cycle of firms. *Mimeo, CEMFI*.
- Lee, H. (2023). Striking while the iron is cold: Fragility after a surge of lumpy investments. *Mimeo, University of Tokyo*.
- Loria, F., C. Matthes, and D. Zhang (2023). Assessing macroeconomic tail risk. *Available at SSRN* 4002665.

- Mertens, K. and M. O. Ravn (2013). The dynamic effects of personal and corporate income tax changes in the united states. *American Economic Review* 103(4), 1212–47.
- Ottobello, P. and T. Winberry (2020). Financial heterogeneity and the investment channel of monetary policy. *Econometrica* 88(6), 2473–2502.
- Ramey, V. A. and S. Zubairy (2018). Government spending multipliers in good times and in bad: evidence from US historical data. *Journal of Political Economy* 126(2), 850–901.
- Rauh, J. (2006). Investment and Financing Constraints: Evidence from the Funding of Corporate Pension Plans. *The Journal of Finance* 61(1), 33–71.
- Reiter, M., T. Sveen, and L. Weinke (2013). Lumpy investment and the monetary transmission mechanism. *Journal of Monetary Economics* 60(7), 821–834.
- Reiter, M., T. Sveen, and L. Weinke (2020). Agency costs and the monetary transmission mechanism. *B.E. Journal of Macroeconomics* 20(1), 1–11.
- Tenreyro, S. and G. Thwaites (2016). Pushing on a string: US monetary policy is less powerful in recessions. *American Economic Journal: Macroeconomics* 8(4), 43–74.
- Thomas, J. K. (2002). Is lumpy investment relevant for the business cycle? *Journal of Political Economy* 110(3), 508–534.
- Winberry, T. (2021). Lumpy investment, business cycles, and stimulus policy. *American Economic Review* 111(1), 364–96.
- Xu, Q. and E. Zwick (2021). Tax policy and abnormal investment behavior. *The Journal of Finance*.
- Zwick, E. and J. Mahon (2017). Tax Policy and Heterogeneous Investment Behavior. *American Economic Review* 59(3), 379–388.

# Online Appendix to “Monetary policy, firm heterogeneity, and the distribution of investment rates”

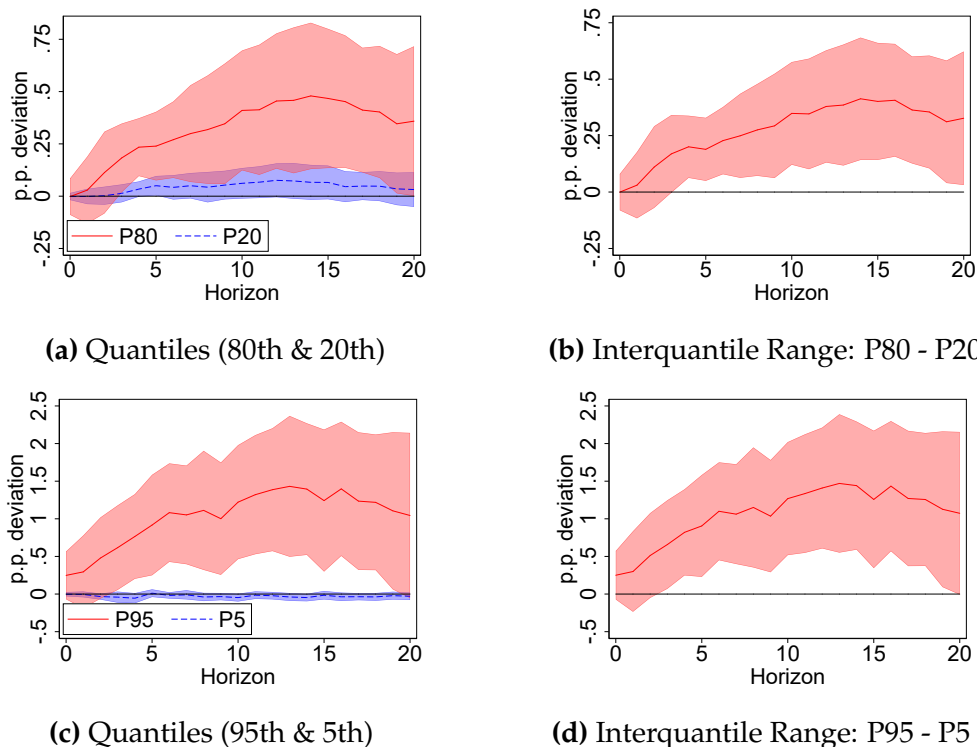
*Matthias Gnewuch, Donghai Zhang*

## A Additional Tables and Figures

Table A.1: Fixed Parameters

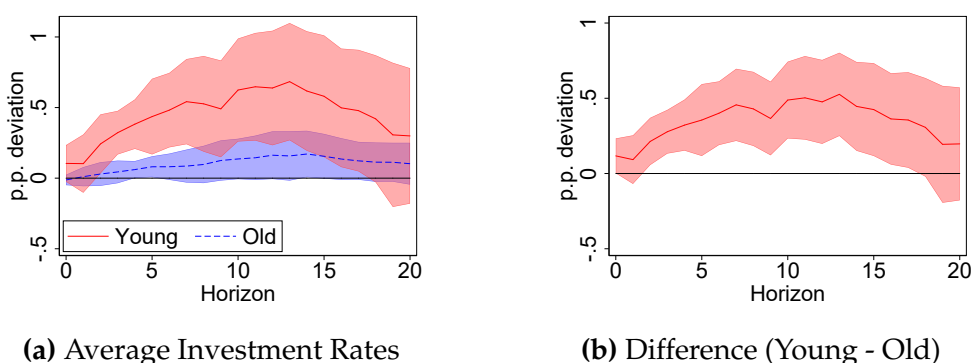
| Parameter                  | Description  | Value |
|----------------------------|--|-------|
| <b>Household</b>           |  |       |
| $\beta$                    | Discount factor                                    | 0.99  |
| $\psi$                     | Labour Disutility                                  | 0.55  |
| <b>Investment Block</b>    |  |       |
| $\theta$                   | Capital Coefficient                                | 0.21  |
| $\nu$                      | Labour Coefficient                                 | 0.64  |
| $\delta$                   | Depreciation Rate                                  | 1.93% |
| $\rho_z$                   | Persistence of TFP Shock                           | 0.95  |
| $\pi^{exit}$               | Exit Probability                                   | 1.63% |
| <b>New Keynesian Block</b> |  |       |
| $\varphi$                  | Price Adjustment Cost                              | 90    |
| $\gamma$                   | Elasticity of Substitution over Intermediate Goods | 10    |
| $\varphi\pi$               | Taylor Rule Coefficient on Inflation               | 1.5   |
| $\rho_r$                   | Interest Rate Smoothing                            | 0.75  |
| $\kappa$                   | External Capital Adjustment Costs                  | 11    |

**Figure A.1:** Effect of Monetary Policy on Quantiles of the Investment Rate Distribution



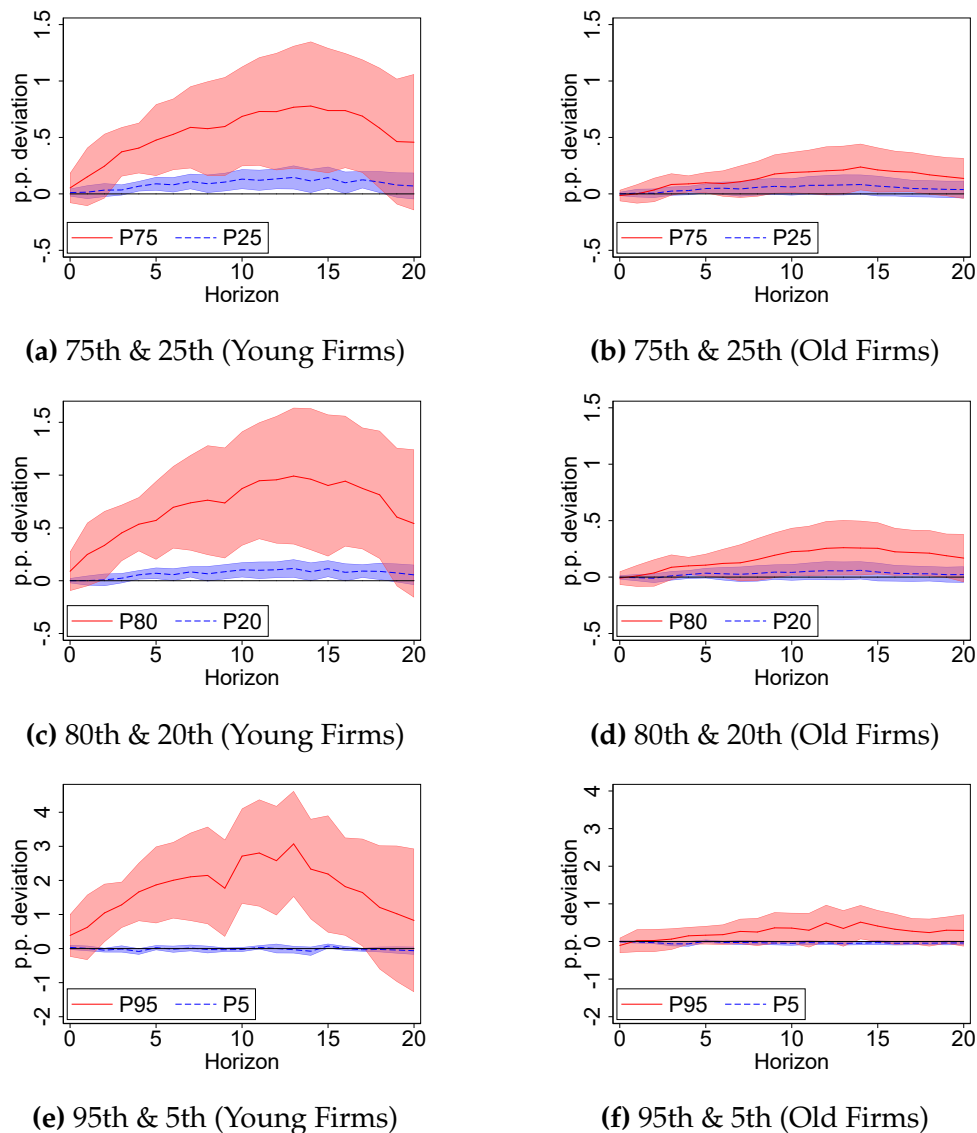
*Notes:* This figure plots the effect of a monetary policy shock on statistics of the investment rate distribution. The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

**Figure A.2:** Effect of Monetary Policy on Age-Group-Specific Average Investment Rates



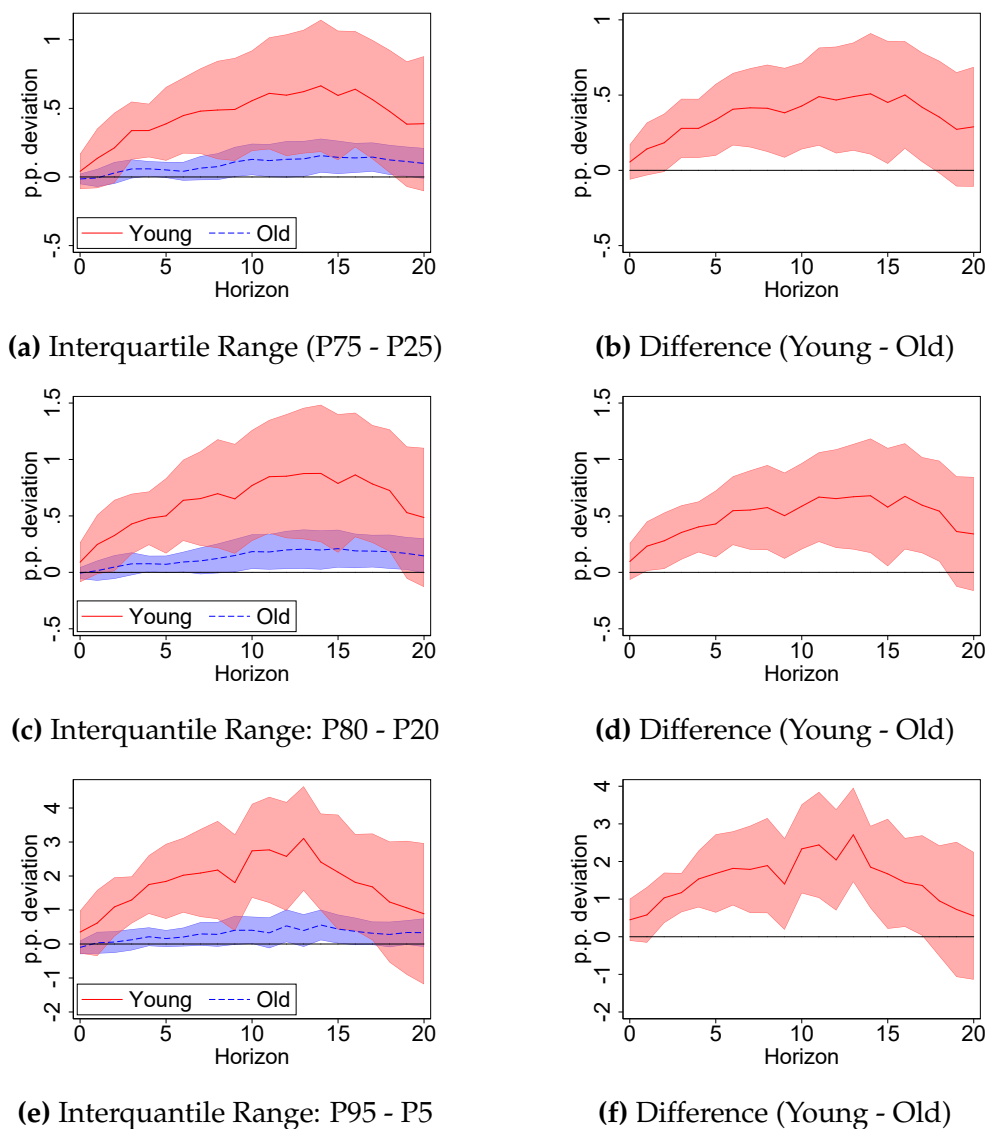
*Notes:* Young (old) firms are less (more) than 15 years old. The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

**Figure A.3: Effect on Quantiles of Age-Group-Specific Inv. Rate Distributions**



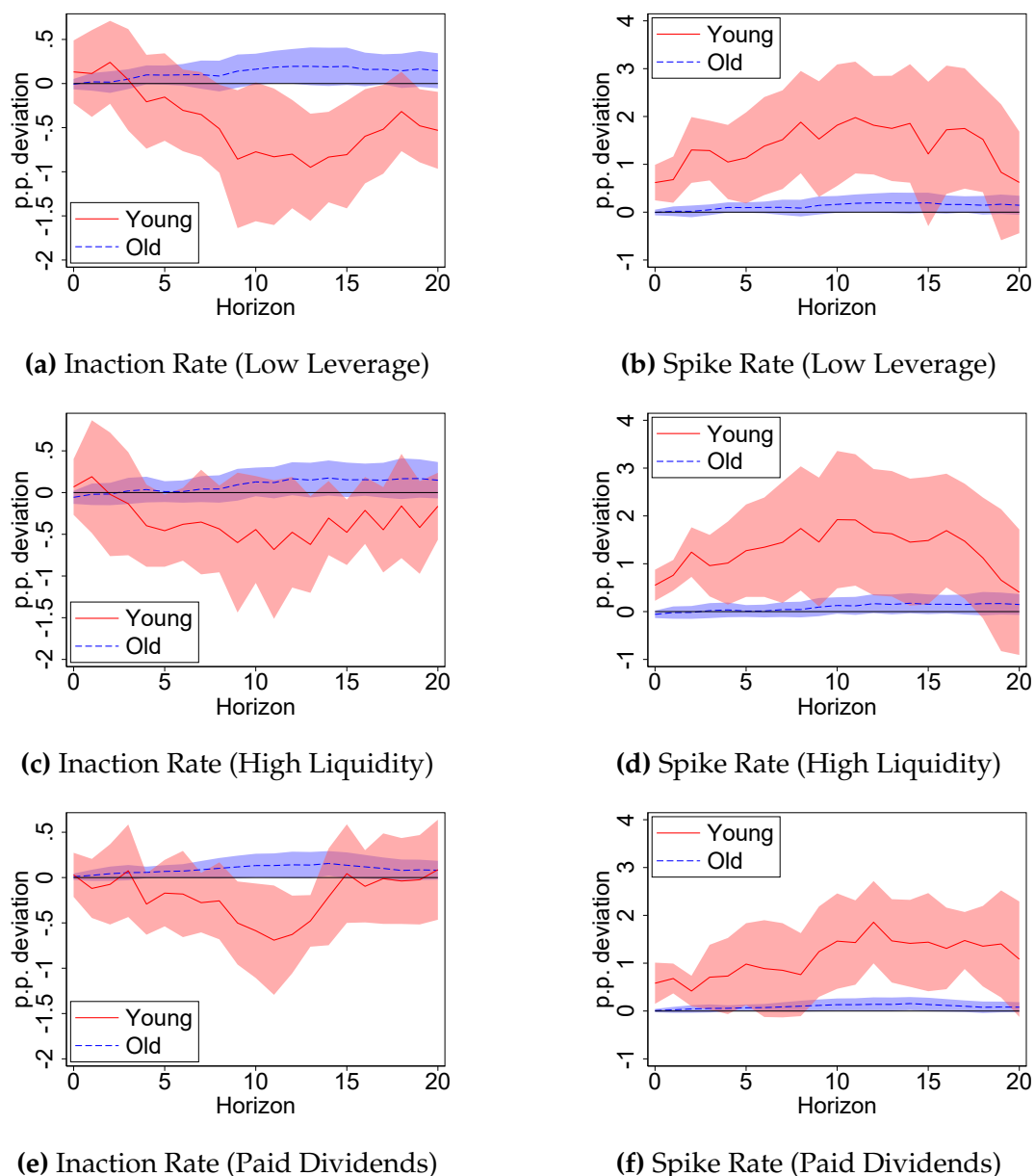
*Notes:* This figure plots the effect of a monetary policy shock on quantiles of the age-specific investment rate distributions. Young (old) firms are firms less (more) than 15 years old. The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

**Figure A.4:** Effect on Interquartile Ranges of Age-Group-Specific Inv. Rate Distributions



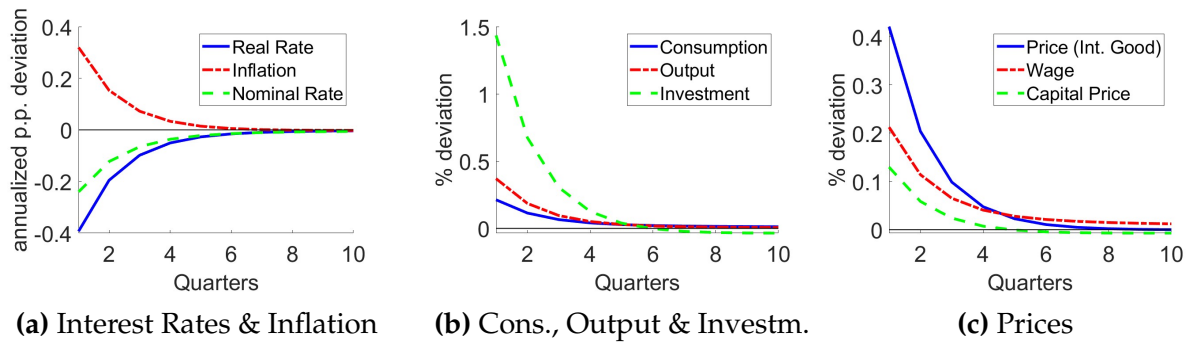
*Notes:* This figure plots the effect of a monetary policy shock on statistics of the age-specific investment rate distributions. Young (old) firms are firms less (more) than 15 years old. The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

**Figure A.5:** Effect of Monetary Policy on Age-Specific Spike & Inaction Rates (Conditional on Markers of Low Financial Constraints)



*Notes:* This figure plots the effect of a monetary policy shock on the spike rate and the inaction rate of young and old firms. Young (old) firms are less (more) than 15 years old. A spike is an investment rate exceeding 10%, inaction is an investment rate less than 0.5% in absolute value. Panels (a) and (b) only use firm observations which were in the bottom tercile of the leverage distribution in the past quarter, panels (c) and (d) uses observations which were in the top tercile of the liquidity distribution in the past quarter, and panels (e) and (f) use only firm observations which have ever paid dividends in the past. The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h e_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

**Figure A.6:** Aggregate Effects of an Expansionary Monetary Policy Shock



*Notes:* This figure plots the effects of a monetary policy shock on interest rates, inflation, aggregates, and prices in the calibrated model.



## B Simple Model

In the main text, we build a heterogeneous-firm life cycle model with capital adjustment costs and nominal rigidities. The purpose is to explain the observed (heterogeneous) effects of interest rate changes on the distribution of investment rates and study their aggregate implications. In the current section, we illustrate the mechanisms at work through the lens of a simple two-period model. Most importantly, the model features fixed capital adjustment costs which create an extensive margin investment decision.

In this simple model, we compare small and large firms. Since age and size are strongly correlated both in the data and in the quantitative model, all intuitions we provide in the simple model hold true when comparing young and old firms in the quantitative model. In Appendix C, we compare the heterogeneous sensitivity by age and by size in the data and in the quantitative model.

The simple model consists of two periods. In period 0, firms are endowed with  $k_0$  units of capital and choose the next period's capital  $k_1$ . The price of one unit of capital relative to the price of the consumption good is  $q$ . In period 1, firms transform capital into the consumption good ( $y$ ) using the decreasing returns to scale production technology  $y = k_1^\theta$  with  $\theta < 1$ . Sales are discounted at the real interest rate  $r$ , and capital depreciates fully during production.

In the absence of adjustment costs, the firms' profit-maximization problem is

$$\max_{k_1} \frac{1}{1+r} k_1^\theta - q(k_1 - k_0). \quad (34)$$

From the first-order condition for  $k_1$ , we obtain the optimal amount of capital that the firm chooses for period 1

$$k_1^* = \left( \frac{\theta}{(1+r)q} \right)^{\frac{1}{1-\theta}} \quad (35)$$

and the optimal (gross) investment rate as a function of firm size  $i^*(k_0) = \frac{k_1^*}{k_0}$ .

We now introduce some features from the quantitative model. First, there is a unit mass of firms within each size category  $k_0$  and firms are indexed by  $j$ . Second, adjusting the stock of capital is subject to a fixed adjustment cost  $\zeta_j \in [0, \bar{\zeta}]$ , which is drawn from a uniform distribution. Moreover, we assume that the economy is populated by firms whose initial capital stocks are below the desired level, i.e.,  $k_{j,0} < k_1^*, \forall k_0$ .<sup>1</sup>

The optimization problem of a firm  $j$  with an initial stock of capital  $k_0$  has changed

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<sup>1</sup>In the steady state of the quantitative model, there are also some firms with capital stocks above their desired level. However, quantitatively, these firms play a minor role.

to:

$$\max_{k_{1,j}} \frac{1}{1+r} k_{1,j}^\theta - q(k_{1,j} - k_0) - \xi_j \mathbb{1}\{k_{1,j} \neq k_0\}, \quad (36)$$

where  $\mathbb{1}\{k_{1,j} \neq k_0\}$  is an indicator variable that equals 1 if  $k_{1,j} \neq k_0$  and 0 otherwise. To solve this problem, let  $VA(k_0)$  denote the value added of adjusting capital while ignoring the fixed adjustment cost:

$$VA(k_0) = \frac{1}{1+r} k_1^{*\theta} - q(k_1^* - k_0) - \frac{1}{1+r} k_0^\theta, \quad (37)$$

where  $k_1^*$  is the optimal amount of capital that firms will acquire conditional on adjusting as defined by equation (35).

Considering the adjustment cost, a firm  $j$  adjusts capital if and only if the value added exceeds the costs, i.e.,  $VA(k_0) > \xi_j$ . The threshold value of  $\xi_j$ , which makes a firm indifferent between adjusting or not, is defined by  $\xi^T(k_0) \equiv VA(k_0)$ . This implies a cutoff rule, i.e., a firm  $j$  will adjust its capital stock if and only if  $\xi_j < \xi^T(k_0)$ . From equation (37), it is evident that this cutoff value not only depends on the initial size of the firm but also on the interest rate  $r$  and the other parameters of the model.

The average investment rate among firms of size  $k_0$  is:

$$\bar{i}(k_0) = \lambda(k_0) \times i^*(k_0) \quad (38)$$

where  $\lambda(k_0) = \frac{\xi^T(k_0)}{\bar{\xi}} \in [0, 1]$  denotes the share of firms of size  $k_0$  that choose to invest, i.e. the hazard rate. Conditional on investing, firms choose the optimal investment rate  $i^*(k_0)$  as defined above.

The group-specific interest rate sensitivity of the investment rate is:

$$\frac{\partial \bar{i}(k_0)}{\partial r} = \underbrace{\frac{\partial \lambda(k_0)}{\partial r} i^*(k_0)}_{\text{Extensive Margin}} + \underbrace{\lambda(k_0) \frac{\partial i^*(k_0)}{\partial r}}_{\text{Intensive Margin}}, \quad (39)$$

which features two components. There is an intensive margin effect,  $\lambda(k_0) \frac{\partial i^*(k_0)}{\partial r}$ , because firms that would be adjusting anyways choose a different investment rate. Moreover, there is an extensive margin effect,  $\frac{\partial \lambda(k_0)}{\partial r} i^*(k_0)$ , because more or less firms choose to invest at all. Motivated by our empirical findings, this paper emphasises the extensive margin effect.

Proposition 1 provides the main theoretical findings of this paper, which regard the effect of interest rate changes on the hazard rate ( $\frac{\partial \lambda(k_0)}{\partial r}$ ) as well as how the sensitivity of the average investment rate due to the extensive margin changes with firm size.

**Proposition 1.** *In an economy populated by heterogenous-firms that face fixed adjustment costs as described above, it holds that*

1. *An interest rate cut increases the hazard rate:  $\frac{\partial \lambda(k_0)}{\partial r} < 0$*
2. *The sensitivity of the average investment rate to interest rate changes via the extensive margin is decreasing (in absolute terms) in firm size:  $\frac{\partial \left( \frac{\partial \lambda(k_0)}{\partial r} i^*(k_0) \right)}{\partial k_0} > 0$*

*Proof.* See Appendix B.1. □

The first part of Proposition 1 establishes that an interest rate cut increases the hazard rate in line with the empirical evidence shown in Figure 6. The costs of investing (cost of additional capital, adjustment cost) are paid in period 0, whereas the benefits materialize in period 1. When the interest rate falls, the discounted benefit of investing rises. Hence, the value added of adjusting and thus the hazard rate rise.<sup>2</sup>

Figure B.1a provides visual intuition by plotting the value added for a given  $k_0$ ,  $VA(k_0)$ , against the random fixed cost  $\zeta$ . The black upward-sloping line is the 45° line indicating the points where VA equals  $\zeta$ . The intercept of the two curves pins down the cutoff value  $\zeta^T$ . The green dotted line plots the density function of  $\zeta$  (uniform distribution). The area under the density function to the left of the cutoff value  $\zeta^T$  is the mass of adjusting firms. An interest rate cut shifts the VA curve upwards. As a result, the cutoff value  $\zeta^T$  increases and so does the mass of adjusting firms as indicated by the green shaded area.

The second part of Proposition 1 establishes that the effect of an interest rate cut on the group-specific average investment rate via the extensive margin is larger among small firms. To understand this result, it is useful to compare the extensive margin effect for groups of small (S) and large (L) firms:

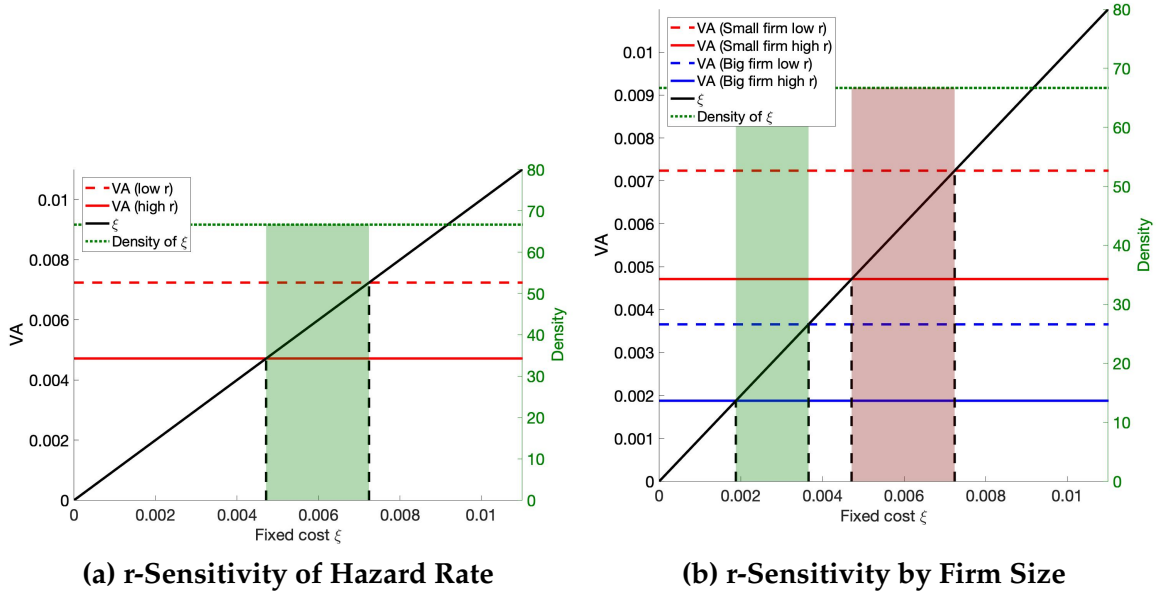
$$\begin{aligned}
 \text{HetExt}_{S-L} &= \underbrace{\frac{\partial \lambda(k_{0,S})}{\partial r} i^*(k_{0,S})}_{\text{Small Firms}} - \underbrace{\frac{\partial \lambda(k_{0,L})}{\partial r} i^*(k_{0,L})}_{\text{Large Firms}} \\
 &= \underbrace{\frac{\partial \lambda(k_{0,L})}{\partial r} (i^*(k_{0,S}) - i^*(k_{0,L}))}_{\text{Heterogeneous Size Effect}} + \underbrace{\left( \frac{\partial \lambda(k_{0,S})}{\partial r} - \frac{\partial \lambda(k_{0,L})}{\partial r} \right) i^*(k_{0,S})}_{\text{Heterogeneous Hazard Rate Increase}} \quad (40)
 \end{aligned}$$

This decomposition shows that there are two mechanisms. First, there is the *heterogeneous size effect*, due to which even if an interest rate cut had the same effect on hazard rates of small and large firms, there would be a differential effect on average

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<sup>2</sup>In the quantitative model, there are of course additional effects, but the main intuition – an interest rate cut raising the value added of investing – remains the same.

**Figure B.1: Intuition for Proposition 1**



*Notes:* This figure plots the value added of investing (VA) of a firm against the random fixed cost  $\zeta$ . The black upward-sloping line is the 45° line indicating the points where VA equals  $\zeta$ . The intercept of the two curves pins down the threshold value of  $\zeta^T$ . The green dotted line plots the density function of  $\zeta$  (uniform distribution). The area under the density function to the left of the threshold value  $\zeta^T$  is the hazard rate. The shaded area in Panel (a) plots the difference in the hazard rate after an interest rate change. Panel (b) plots the difference in the hazard rate for a small and a big firm.

investment rates. This is because among the *new* adjusters, small firms have higher investment rates conditional on adjusting ( $i^*(k_{0,S}) - i^*(k_{0,L}) > 0$ ). This follows from the observation that in this simple model, conditional on investing, all firms choose  $k_1^*$  and the investment rate is defined by  $i^* = \frac{k_1^*}{k_0}$ . In the absence of an extensive margin investment decision, this effect would disappear because  $\frac{\partial \lambda(k_0)}{\partial r} = 0$ .

Second, interestingly, an interest rate cut increases the hazard rate of small firms by more than the hazard rate of large firms. This result aligns well with the empirical evidence that the spike rate of small (young) firms reacts more strongly to a monetary shock than the spike rate of large (old) firms (see Figure C.2 for size and Figure 6 for age). As discussed above, the hazard rate rises, because the value added of investing rises, which happens because the discounted benefit of investing rises. This increase in the discounted benefit of investing is larger for small firms. The reason for this is that small firms have a higher marginal product of capital because of decreasing returns to scale. Hence, the interest rate cut has a larger effect on the hazard rate of small firms.

Figure B.1b provides visual intuition for the heterogeneous effect of an interest rate cut on hazard rates. The cut in the interest rate shifts the VA of small firms (red lines)

up by more than the VA of big firms (blue lines). As a result, the change in the hazard rate is more pronounced for small firms (red-shaded area) than for big firms (green-shaded area).

To sum up, we have highlighted two effects in this simple model. First, an interest rate cut increases the hazard rate, i.e. the fraction of firms deciding to make an investment. Therefore, a change in the interest rate changes the distribution of investment rates. Second, the average investment rate of small firms responds more strongly along the extensive margin to interest rate changes than the average investment rate of large firms.

Regarding the second effect, it is worth pointing out that small firms are more sensitive to interest rate changes *in the absence* of a financial accelerator mechanism. The basic idea of such a mechanism is that interest rate changes affect financing conditions and small firms are more exposed to financing conditions than large firms. Then, interest rate changes have a heterogeneous effect on investment because there is a heterogeneous effect on the *cost* of investing, as e.g. in [Ottonello and Winberry \(2020\)](#). In contrast, in this model, there is a heterogeneous effect of interest rate changes on investment because of a heterogeneous effect on the *benefit* of investing.<sup>3</sup> This is because small firms have a higher marginal product of capital.

## B.1 Proofs

**Proposition 1.** *In an economy populated by heterogenous-firms that face fixed adjustment costs as described above, it holds that*

1. *An interest rate cut increases the hazard rate:  $\frac{\partial \lambda(k_0)}{\partial r} < 0$*
2. *The sensitivity of the average investment rate to interest rate changes via the extensive margin is decreasing (in absolute terms) in firm size:  $\frac{\partial \left( \frac{\partial \lambda(k_0)}{\partial r} i^*(k_0) \right)}{\partial k_0} > 0$*

*Proof.* Rearranging equation (37), the value added of adjusting capital while ignoring the fixed adjustment cost is:

$$VA(k_0) = \frac{1}{1+r} \left( k_1^{*\theta} - k_0^\theta \right) - q(k_1^* - k_0) \quad (41)$$

where  $k_1^*$  was defined in equation (35). Using the definition of the cutoff  $\xi^T(k_0)$  and

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<sup>3</sup>Even though the capital adjustment costs that we impose can in principle be interpreted as stand-ins for financial frictions, the model does not feature a financial accelerator mechanism. This is because by construction, the capital adjustment costs are themselves not affected by aggregate shocks, including monetary policy shocks.

the hazard rate  $\lambda(k_0)$  from the main text, we have

$$\lambda(k_0) = \frac{1}{\bar{\xi}} VA(k_0). \quad (42)$$

Taking the derivative w.r.t. the real interest rate, we get

$$\frac{\partial \lambda(k_0)}{\partial r} = -\frac{1}{\bar{\xi}} \frac{1}{(1+r)^2} (k_1^{*\theta} - k_0^\theta) < 0, \quad (43)$$

which proves the first part of the proposition. Note that  $k_0 < k_1^*$  by assumption.

The second part of the proposition requires

$$\frac{\partial \left( \frac{\partial \lambda(k_0)}{\partial r} i^*(k_0) \right)}{\partial k_0} = \frac{\partial^2 \lambda(k_0)}{\partial r \partial k_0} i^*(k_0) + \frac{\partial \lambda(k_0)}{\partial r} \frac{\partial i^*(k_0)}{\partial k_0} > 0. \quad (44)$$

The first term is positive, because

$$\frac{\partial^2 \lambda(k_0)}{\partial r \partial k_0} = \frac{1}{\bar{\xi}} \frac{1}{(1+r)^2} \theta k_0^{\theta-1} > 0 \quad (45)$$

and  $i^*(k_0) > 0$  because  $k_0, k_1 > 0$ . The second term is positive because

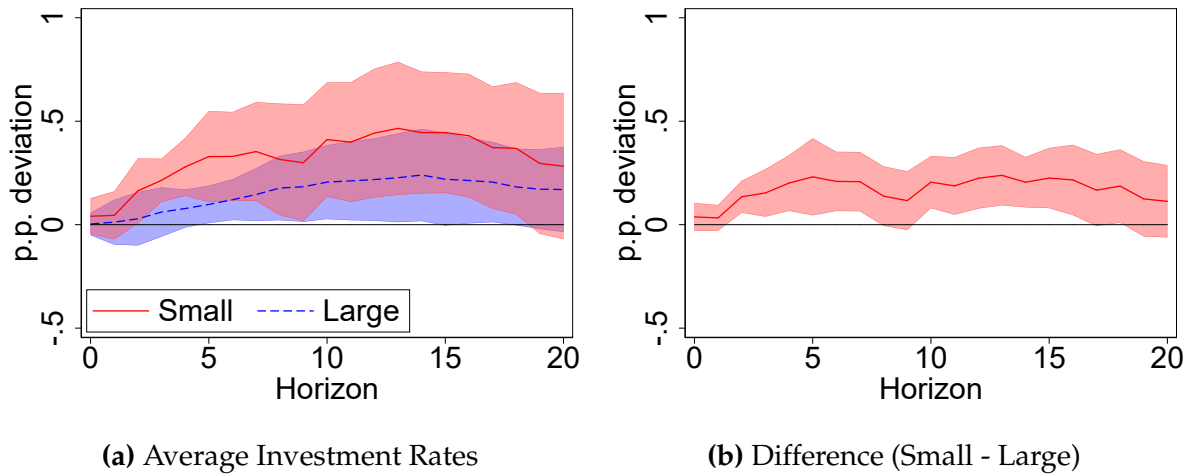
$$\frac{\partial i^*(k_0)}{\partial k_0} = -k_1^* k_0^{-2} < 0 \quad (46)$$

and  $\frac{\partial \lambda(k_0)}{\partial r} < 0$  as shown in equation (43). Thus, the inequality in equation (44) holds which completes the proof.  $\square$

## C Heterogeneous Effects by Firm Size

**Empirical Evidence** Cloyne et al. (2023) have shown that being young is a better predictor of a firm's sensitivity to monetary policy shocks than being small. We replicate this finding in Figure C.1. Firms that are smaller than the median are at the peak on average 24 basis points more sensitive than firms which are larger than the median. In comparison, young firms are at the peak on average 53 basis points more sensitive than old firms, as shown in Figure A.2. This weaker heterogeneous effect goes along with a weaker heterogeneous effect along the extensive margin, as shown in Figure C.2, which replicates Figure 6 while grouping firms by size instead of age. In addition, the change in the distribution differs somewhat less across size groups than across age groups, as can be seen from comparing Figure A.3 with Figure C.4.

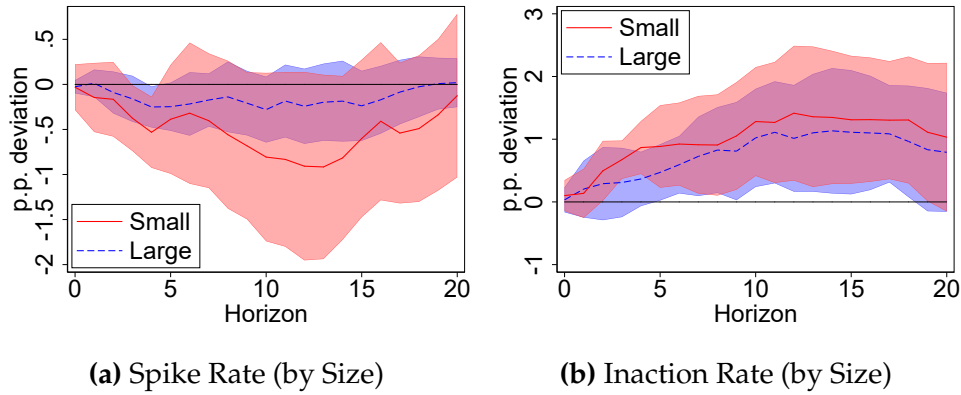
**Figure C.1:** Effect of Monetary Policy Shock on Average Investment Rates by Size



*Notes:* Small (large) firms are firms smaller (larger) than the median in a given quarter. The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h e_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

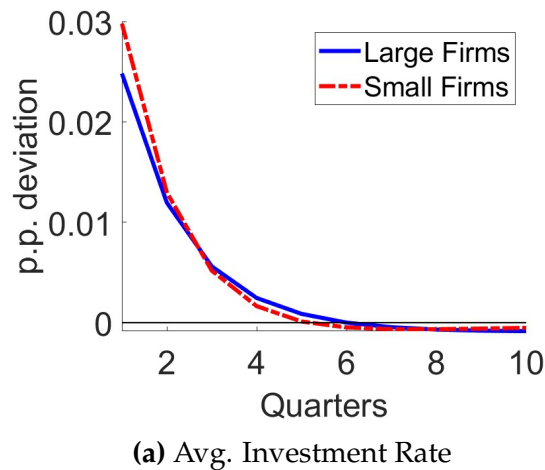
**Model Predictions** Our model is able to replicate the finding that young age is a better predictor of firms' sensitivity to monetary policy shocks than small size. This is evident from Figure C.3, which replicates Figure 14, panel (a), while grouping firms by size instead of age. Firms that are smaller than the median are on impact more sensitive than firms larger than the median, but the difference is substantially smaller than the gap between young and old firms. Intuitively, age is the better predictor of sensitivity, because young firms are more likely to be "close to making a large investment". This is because young firms are born small and will almost certainly grow in the future. In contrast, small firms may or may not be "close to making a large investment". This is because some firms are small because they are very unproductive, such that the low level of capital is their desired level of capital. In a nutshell, size correlates positively with productivity, while age is uncorrelated with productivity in our model.

**Figure C.2: Effect on Group-Specific Spike & Inaction Rates (by Size)**



*Notes:* This figure plots the effect of a monetary policy shock on the spike rate and the inaction rate of small and large firms. Small (large) firms are firms smaller (larger) than the median in a given quarter. A spike rate is an investment rate exceeding 10%, an inaction rate is an investment rate less than 0.5% in absolute value. The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

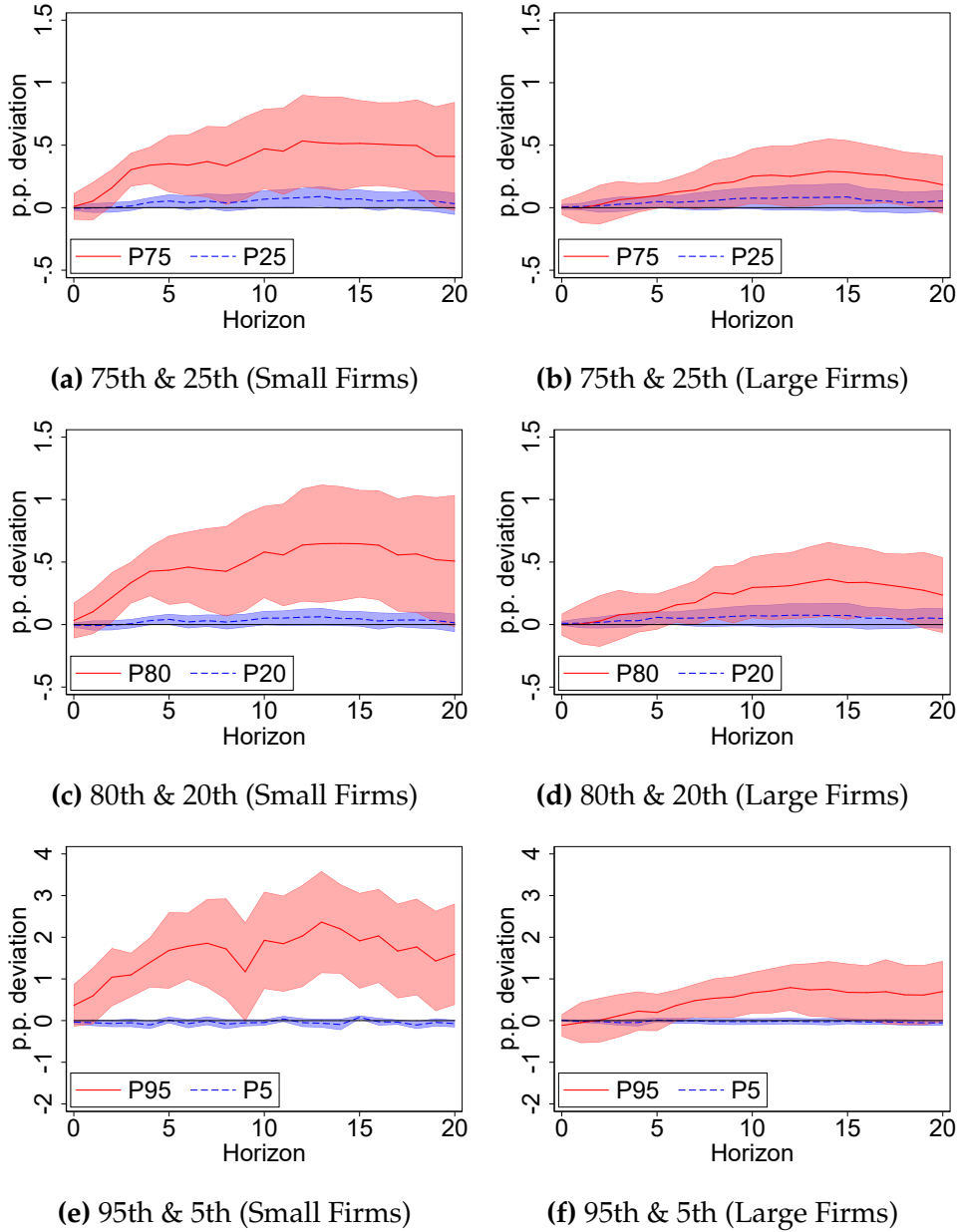
**Figure C.3: Heterogeneous Effect (by Size Group) of an Exp. Monetary Policy Shock**



*Notes:* This figure plots the effect of a monetary policy shock on the average investment rates of small and large firms in the model. Small (large) firms are firms smaller (larger) than the median in a given quarter.



**Figure C.4:** Effect on Quantiles of Size-Group-Specific Inv. Rate Distributions



*Notes:* This figure plots the effect of a monetary policy shock on quantiles of the size-specific investment rate distributions. Small (large) firms are firms smaller (larger) than the median in a given quarter. The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

## D Data Appendix

### D.1 Sample Selection

We use the Compustat North America Fundamentals Quarterly database. Observations are uniquely identified by GVKEY & DATADATE. In line with the literature, we exclude observations which fall under the following criteria

1. not incorporated in the United States (based on FIC)
2. native currency not U.S. Dollar (based on CURNCDQ)
3. fiscal quarter does not match calendar quarter (based on FYR)
4. specific sectors
  - Utilities (SIC 4900-4999)
  - Financial Industry (SIC 6000-6999)
  - Non-operating Establishments (SIC 9995)
  - Industrial Conglomerates (SIC 9997)
  - Non-classifiable (NAICS > 999900)
5. missing industry information (SIC or NAICS code)
6. missing capital expenditures (based on CAPX)
7. missing or non-positive total assets (AT) or net capital (PPENT)
8. negative sales (SALEQ)
9. acquisitions (based on AQCY) exceed 5% of total assets (in absolute terms)
10. missing or implausible age information (see Appendix [D.2](#))
11. outlier in the Perpetual Inventory Method (see Appendix [D.3](#))

Our sample begins with 1986Q1 and ends with 2018Q4. In a final step, we exclude firm which we observe for less than 20 quarters, unless they are still in the sample in the final period. This ensures that we do not mechanically exclude all firms incorporated in the last five years of our sample.

## D.2 Firm Age

We use data on firm age from WorldScope and Jay Ritter’s database<sup>4</sup>. WorldScope provides the date of incorporation (Variable: INCORPDATE), while Jay Ritter’s database provides the founding date. Both are merged with Compustat based on CUSIP. We define as the firm entry quarter the minimum of both dates if both are available. We do not use information on the initial public offering (IPO) of a firm to determine its age, since the time between incorporation and IPO can vary substantially. However, we use the IPO date to detect implausible age information. We exclude firms for which the IPO date reported in Compustat (IPODATE) precedes the firm entry quarter by more than four quarters. In similar fashion, we exclude firms which appear in Compustat more than four quarters before the firm entry quarter.<sup>5</sup> Finally, we merge information on the beginning of trading from CRSP (Variable: BEGDAT) based on CUSIP and likewise exclude firms with trading more than four quarters before the firm entry quarter.

## D.3 Perpetual Inventory Method

Accounting capital stocks  $k_{j,t}^a$  as reported in Compustat deviate from *economic* capital stocks for at least two reasons. First, accounting depreciation is driven by tax incentives and usually exceeds economic depreciation. Second, accounting capital stocks are reported at historical prices, not current prices. With positive inflation, both issues make the economic capital stock exceed the accounting capital stock. Therefore, we use a Perpetual Inventory Method (PIM) to compute real economic capital stocks, building on [Bachmann and Bayer \(2014\)](#).

**Investment.** In principle, there are two options to measure net nominal quarterly investment. First, investment can be measured directly ( $I_{j,t}^{dir}$ ) from the Statement of Cash Flows as capital expenditures (CAPX) less the sale of PPE (SPPE)<sup>6</sup>. Second, investment can be backed out ( $I_{j,t}^{indir}$ ) from the change in PPE (D.PPENT) plus depreciation (DPQ), using Balance Sheet and Income Statement information. Either measure needs to be deflated to obtain real investment. We use INVDEF from FRED, which has the advantage of being quality-adjusted. We prefer the direct investment measure, since the indirect measure basically captures any change to PPE, including changes due to acquisitions. Nevertheless, we want to exclude observations where both in-

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<sup>4</sup><https://site.warrington.ufl.edu/ritter/>

<sup>5</sup>We do not construct firm age from the first appearance in Compustat. An inspection of the data reveals that this would result in wrongly classifying a number of old and established firms as young. [Cloyne et al. \(2023\)](#) do so but show in an earlier working paper version that results are unchanged if only age information from WorldScope is used.

<sup>6</sup>We follow [Belo et al. \(2014\)](#) and set missing values of SPPE to zero.

vestment measures differ strongly. To this end, we compute investment rates using lagged net accounting capital (L.PPENT), compute the absolute difference between both and discard the top 1% of that distribution.

**Depreciation Rates.** We obtain economic depreciation rates from the Bureau of Economic Analysis' (BEA) Fixed Asset Accounts. Specifically, we retrieve current-cost net stock and depreciation of private fixed assets by year and industry.<sup>7</sup> We calculate annual depreciation rates by industry and assume a constant depreciation rate within the calendar year to calculate quarterly depreciation rates.

**Real Economic Capital Stocks.** We initialise a firm's capital stock with the net (real) accounting capital stock  $k_{j,1}^a$  (PPENT / INVDEF) whenever this variable is first observed. We iterate forward using deflated investment and the economic depreciation rate.

$$k_{j,1}^{(1)} = k_{j,1}^a \quad (47)$$

$$k_{j,t+1}^{(1)} = (1 - \delta_t^e)k_{j,t}^{(1)} + \frac{p_t^I}{p_{2009,t}} I_{j,t}^{dir} \quad (48)$$

Comparing  $k_{j,t}^{(1)}$  and  $k_{j,t}^a$  shows non-negligible discrepancies. On average, the economic capital stock is larger, confirming the hypothesis that accounting capital stocks are understated. This makes it problematic to use the accounting capital stock as a starting value in the PIM. As a remedy, we again follow [Bachmann and Bayer \(2014\)](#) and use an iterative procedure to re-scale the starting value. We compute a time-invariant scaling factor  $\phi$  at the sector-level and use it to re-scale the starting value as follows. We iterate until  $\phi$  converges. The procedure is initialized with  $k_{j,t}^{(0)} = k_{j,t}^a$  and  $\phi^{(0)} = 1$ .

$$\phi^{(n)} = \frac{1}{NT} \sum_{j,t} \frac{k_{j,t}^{(n)}}{k_{j,t}^{(n-1)}} \quad [\text{and not in top or bottom 1\%}] \quad (49)$$

$$k_{j,1}^{(n+1)} = \phi^{(n)} k_{j,1}^{(n)} \quad (50)$$

**Outliers.** We exclude firms for which the economic capital stock becomes negative at any point in time. This can arise if there is a sale of capital which exceeds current economic capital. Further, we compute the deviation between (real) accounting and economic capital stocks and discard the top 1% of that distribution. Finally, we discard

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<sup>7</sup>The Fixed Asset Accounts also provide depreciation rates by asset type (Equipment, Structures, Intellectual Property Products), which we do not use since the firm-level data does not include information on capital stocks or capital expenditure by asset type.

firms for which we have less than 20 observations, unless they are still in the sample in the final quarter.

**Evaluation.** Our estimated real economic capital stock is still highly correlated with the real accounting capital stock. A simple regression has an  $R^2$  of above 0.96 and shows that the economic capital stock is on average slightly higher (by about 4%), as expected. The investment rate (net real investment over lagged real economic capital) is highly correlated ( $\rho > 0.98$ ) with the accounting investment rate used in [Cloyne et al. \(2023\)](#). A simple regression shows that on average, the economic investment rate is lower (by about 13%) than the accounting investment rate, also as expected due to the underreporting of accounting capital stocks.

## D.4 Variable Construction

Most of our variables follow the definitions in the literature. Our baseline measure of the investment rate is  $i_{jt} = \frac{CAPX_{jt} - SPPE_{jt}}{INVDEF_t \times k_{jt-1}}$ , thus, real capital expenditures (CAPX) net of sales of capital (SPPE) divided by the lagged real economic capital stock, computed as described previously. To measure size, we use the log of total assets (AT).

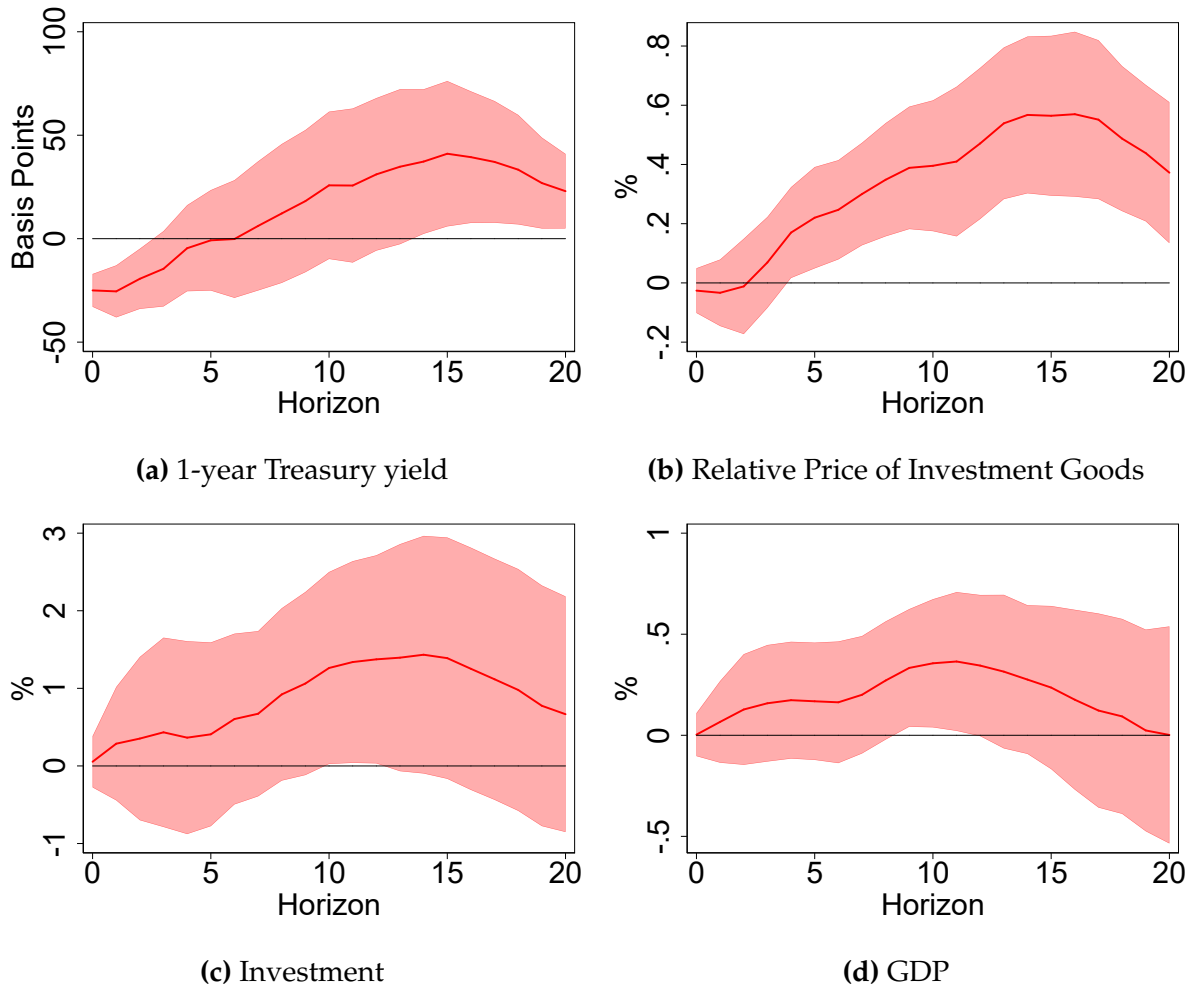
## D.5 Identification of Monetary Policy Shocks

We use the monetary policy shocks implied by the proxy SVAR used in [Gertler and Karadi \(2015\)](#). We calculate them according to the following procedure. First, we update the data used in the [Gertler and Karadi \(2015\)](#) baseline SVAR. They use monthly data from 1979M7 to 2012M6. We update all time series to 2019M12. The SVAR includes (the log of) industrial production (FRED: INDPRO), (the log of) the consumer price index (FRED: CPIAUCSL), the one-year government bond rate (FRED: GS1), and the excess bond premium (Source: [https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/files/ebp\\_csv.csv](https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/files/ebp_csv.csv), retrieved in February 2020). Moreover, we update the instrument (cumulative high-frequency FF4 surprises) to 2015M10. Then, we run the SVAR and compute the implied structural monetary policy shocks. See the appendix of [Mertens and Ravn \(2013\)](#) for details. Importantly, even though the instrument is only available until 2015M10, we can compute the structural monetary policy shock until 2019M12.

## D.6 Effects of Monetary Policy using Aggregate Data

Using time series data from FRED, we document the aggregate effects of the monetary policy shocks we utilise. Qualitatively, these are quite similar to [Gertler and Karadi](#)

**Figure D.1:** Aggregate Effects of a Monetary Policy Shock



Notes: The lines represent the estimated  $\hat{\beta}^h$  from separate regressions:  $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$ . The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4. All variables except for the 1-year Treasury yield are in logs.

(2015). Panel (a) of Figure D.1 shows that a monetary policy shock decreases the 1-year Treasury yield (FRED: GS1) for roughly 4 quarters. Thereafter, it overshoots, as observed in Gertler and Karadi (2015). Panels (b) and (c) show that (real) investment (FRED: PNFI) and the relative price of capital goods (FRED: PIRIC) increase strongly. The peak effect on investment is roughly 1.4%. Panel (d) shows that real GDP (FRED: GDPC1) also increases following an expansionary shock. The peak effect is about 0.35%.

## E Analysis of the Calibrated Model

### E.1 Equilibrium Definition

A recursive competitive equilibrium in this model is a set of value functions  $\{V_t(z, k), CV_t^{exit}(z, k), CV_t^a(z, k, \xi), CV_t^n(z, k)\}$ , policy functions  $\{n_t^*(z, k), k_t^*(z, k, \xi), \xi_t^T(z, k)\}$ , quantities  $\{C_t, Y_t, I_t^Q, K_t, N_t\}$ , prices  $\{p_t, w_t, \pi_t, \Lambda_{t+1}, q_t\}$ , and distributions  $\{\mu_t(z, k)\}$  such that all agents in the economy behave optimally, the distribution of firms is consistent with decision rules, and all markets clear:

1. Investment Block: Taking all prices as given,  $V_t(z, k)$ ,  $CV_t^{exit}(z, k)$ ,  $CV_t^a(z, k, \xi)$ , and  $CV_t^n(z, k)$  solve the Bellman equation with associated decision rules  $n_t^*(z, k)$ ,  $k_t^*(z, k, \xi)$ , and  $\xi_t^T(z, k)$ .
2. Household Block: Taking prices as given,  $C_t$  and  $C_{t+1}$  satisfy the household's optimality conditions (29) and (30).
3. New Keynesian Block: The New Keynesian Phillips Curve holds. The Taylor rule holds. Taking prices a given,  $I_t^Q$  satisfies (25).
4. All markets (final good, capital, labour) clear.
5. The distribution of firms,  $\mu_t(z, k)$ , evolves as implied by the decision rules  $k^*(z, k, \xi)$  and  $\xi_t^T(z, k)$ , the exogenous process for firm-level productivity, and considering exogenous exits and entrants with capital  $k_0$  and productivity from  $\mu^{ent}$ .

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