Combining time-variation and mixed-frequencies: an analysis of government spending multipliers in Italy

This paper finds that Italian government spending shocks had a strong impact on GDP in the early 1990s and again during the most recent recession.
Combining time-variation and mixed-frequencies: an analysis of government spending multipliers in Italy*

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Abstract

In this paper, we propose a time-varying parameter vector autoregression (VAR) model with stochastic volatility which allows for estimation on data sampled at different frequencies. Our contribution is two-fold. First, we extend the methodology developed by Cogley and Sargent (2005), and Primiceri (2005), to a mixed-frequency setting. In particular, our approach allows for the inclusion of two different categories of variables [high-frequency and low-frequency] into the same time-varying model. Second, we use this model to study the macroeconomic effects of government spending shocks in Italy over the 1988Q4-2013Q3 period. Italy - as well as most other euro area economies - is characterised by short quarterly time series for fiscal variables, whereas annual data are generally available for a longer sample before 1999. Our results show that the proposed time-varying mixed-frequency model improves on the performance of a simple linear interpolation model in generating the true path of the missing observations. Second, our empirical analysis suggests that government spending shocks tend to have positive effects on output in Italy. The fiscal multiplier, which is maximized at the one year horizon, follows a U-shape over the sample considered: it peaks at around 1.5 at the beginning of the sample, it then stabilizes between 0.8 and 0.9 from the mid-1990s to the late 2000s, before rising again to above unity during the recent crisis.

Key words
Time variation, mixed-frequency data, government spending multiplier

JEL codes
C32, E62, H30, H50

*This version: December 2015; first version: December 2013. We would like to thank the participants at an internal ECB seminar, at the ESCB’s Working Group of Public Finance meeting in Sofia, at the ESM’s Workshop on fiscal multipliers, at the European Economic Association conference in Toulouse and at the Computing in Economics and Finance conference in Oslo, for their useful comments. In particular, we would like to thank Fabio Canova, Sandra Eickmeier, Patrick Fève, Domenico Giannone, Joan Paredes, Vilém Valenta, for helpful suggestions. The opinions expressed herein are those of the authors and do not necessarily reflect those of the the European Central Bank, the Eurosystem and the European Stability Mechanism.

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1 Introduction

In empirical macroeconomic research, vector autoregressions (VARs) are extensively used for forecasting and to analyse the transmission mechanism of structural shocks. Given a certain set of variables of interest, VARs are typically estimated on data aligned at the lowest available frequency, i.e., generally the monthly or quarterly frequency, by taking the average or end-of-period observations of the high frequency variables. In this way, potentially important information from high-frequency dynamics can be lost. In some other cases, the scope for conducting a VAR-based empirical analysis is limited by the unavailability of a high frequency data and/or by a short length of these time series. For example, for euro area countries, most fiscal series are available at an annual frequency prior to 1999 and at a quarterly frequency beginning only in 1999. The lack of sufficiently long quarterly time series has limited the development of the empirical literature on the effects of fiscal policies for these countries.

To address the problem of insufficiently long high-frequency time series, data sampled at different frequencies and, in general, irregular data patterns, a very rich research vein known as “mixed-frequency” literature has surged (see below and Foroni and Marcellino (2013) for an exhaustive survey).

The contribution of this paper is two-fold. First, we add to the mixed-frequency literature by proposing a time-varying parameter VAR model with stochastic volatility which allows for arbitrary data patterns of mixed frequency or irregularly spaced observations. Hereafter, we will refer to this model as TV-MF-BVAR. Indeed, many advanced economies have been characterized by significant sub-sample instability because of several key structural changes experienced over recent decades. For European countries, examples include the adoption of the Maastricht Treaty in 1992, the introduction of the euro in 1999 and the single monetary policy since then, and the recent economic and sovereign debt crisis. Given these structural changes, analysing the transmission of macroeconomic
shocks based on fixed-parameter time series models, and assuming that the variance of shocks has not changed over time, may be inappropriate for these countries.

We extend the methodology proposed by Cogley and Sargent (2005), and Primiceri (2005), on time-varying parameter VARs with stochastic volatility (TV-BVAR) by allowing for the estimation on data sampled at different frequencies. More specifically, our approach allows for the inclusion of two different categories of variables (high-frequency and low-frequency), in the same VAR model without altering the frequency at source by, e.g., taking an ad-hoc data transformation of the high-frequency variables to align them with the low-frequency variables. In the proposed algorithm, the low-frequency variables are treated as high-frequency variables with missing observations. The estimation follows the same steps of Primiceri (2005)’s time-varying VAR, but requires an additional step in order to generate the missing observations for the low-frequency variables. Such missing observations are treated as unobserved state vectors. In this context, the additional step facilitates the extraction of the latent observations for the low frequency variables by means of the Carter and Kohn (1994)’s smoother.

Our second contribution consists of using this model to provide new estimates of the effects of government spending shocks in Italy over the period 1988Q4-2013Q3. We believe that Italy is an interesting case study. The third largest euro zone economy, Italy has experienced a severe economic downturn since 2009, has a fragile fiscal position with a government debt ratio of around 132% of GDP in 2014, and has been at the centre of sovereign market tensions for prolonged periods during the crisis. Italy is also a relevant case for the application of a mixed-frequency approach because, like other European countries, the national statistical agency (ISTAT) only started producing quarterly

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1The literature on VAR models with time-varying parameters and stochastic volatility has expanded in recent years, mainly due to the work of Cogley and Sargent (2005) and Primiceri (2005), which focus on the analysis of monetary policy. D’Agostino et al. (2013) show that time-varying coefficient VAR models with stochastic volatility tend to outperform fixed-coefficient VARs in forecasting GDP and inflation. Canova and Pérez Forero (2014) propose a general framework to estimate a structural VAR that can handle time-varying coefficient or time invariant models, identified with recursive or non-recursive restrictions. More recently, Gali and Gambetti (2015) use a TV-VAR model to study the effects of monetary policy shocks on the U.S. stock market.
time series for most fiscal variables in 1999, though annual time series have generally been available since the 1980s. Indeed, the government spending data used in this paper are published at an annual frequency between 1981 and 1998, and at a quarterly frequency beginning only in 1999Q1. Therefore, the inclusion of both annual and quarterly series for government spending in our model allows us to considerably extend the period covered in the analysis, which would otherwise be limited to the European Economic and Monetary Union (EMU) period, when quarterly data are available.²

Our results indicate that, first, the TV-MF-BVAR model tracks the data in-sample effectively. Second, it accurately produces the path of missing observations. The accuracy, measured by the Mean Squared Errors (MSE) statistics, points to a better performance from the TV-MF-BVAR compared to a linear interpolation approach.

In addition, our empirical analysis suggests that, in Italy, the government spending multiplier reaches its highest values for horizons up to one year after the shock, before declining to zero for longer horizons. The peak fiscal multiplier tends to follow a U-shape: it is around 1.5 between the late 1980s and the early 1990s and it stabilizes at around 0.8 and 0.9 during the run-up phase to the EMU and through the late 2000s. The multiplier then rises again to above unity during the recent global crisis, which is defined here as the period from 2008Q3 (Lehman crisis) until 2013Q3 (end of our sample). At the same time, the average short-term multiplier is around one during the crisis.

The remainder of the paper is organized as follows. Section 2 describes the proposed TV-MF-BVAR model and includes a short review of the literature on mixed-frequency models, Section 3 presents our analysis of the effects of government spending shocks in Italy and Section 4 concludes.

²In our empirical application described in Section 3, the model is used to estimate the missing observations for a low-frequency variable rather than to exploit high frequency data to predict or fit a low-frequency variable (as is often done in the mixed-frequency literature, see e.g. Andreou et al. (2011), Ghysels (2012) and Banbura et al. (2013)). However, our methodology can also be applied when using high-frequency data to “nowcast” a low-frequency variable.
Model

In this Section, we describe our model, which has been built to combine the time-varying framework proposed by Cogley and Sargent (2005) and Primiceri (2005) with a mixed-frequency data environment. In particular, our approach allows for the inclusion of both high-frequency and low-frequency variables in the same TV-BVAR model. In the proposed algorithm, which is described below and in Appendix B, the low-frequency variables are defined as high-frequency variables with missing observations. Such missing observations are treated as unobserved state vectors. In this context, state space models and Bayesian Gibbs sampling provide a natural environment for the estimation of these unobservable state vectors. In particular, we apply the Carter and Kohn (1994)’s algorithm to generate draws for the missing observations at each point in time. Overall, we end up with a model that can accommodate datasets with missing observations and/or unbalanced panel structure due to the different data availability (e.g., ragged data).

2.1 State space representation of the VAR model with missing observations

The model is described as follows. As a first step, the VAR model with missing observations is cast into a state space form. Let us assume that a vector of $N$ endogenous variables $y_t$, eventually sampled at different frequencies with $t$ denoting the highest frequency, can be written as:

\[ y_t = C\tilde{y}_t + v_t \]  

(1)

where $C = I_N$ and $\tilde{y}_t$ is a vector of states, which are known if data are available or unknown otherwise, $v_t \sim N(0, R_t)$ with $R_t$ a $(N \times N)$ diagonal matrix. We denote $r_{i,t}$ as the $i^{th}$ element of the diagonal matrix $R_t$, which can take only two values: 0 if $\tilde{y}_{i,t}$, the $i^{th}$ element of $\tilde{y}_t$, is available, $\infty$ otherwise.

We assume that one variable is available at the low-frequency in the first part of the sample, until
τ, and at the high frequency in the second part of the sample (from τ until T).\(^3\) All other variables are available at the high frequency for the full sample.\(^4\) The algorithm will treat the low-frequency variable as an unobserved state. The missing observations, until τ, will be generated by applying the standard Carter and Kohn (1994)’s recursion, in which we first run the Kalman filter to store the vector mean and the covariance matrix in the last point of the recursion (at time T), we then run the backward recursion to generate draws of \(\tilde{y}_t\). However, under the assumptions of equation 1, it turns out that if a data point at time \(t\) for variable \(i\) is available, then \(\tilde{y}_{i,t}\) is observable. In this case, the corresponding \(i\) row and \(i\) column in the covariance matrix of the normal distribution are zero. The algorithm will then reconstruct exactly the observable as the sum of two components: the conditional expectation of the variable at time \(t\) (using only information until time \(t-1\)) and the prediction error, whose associated Kalman gain, in this particular case, is equal to one. Once this step is finalized, the algorithm foresees the same steps as in Primiceri (2005) for the Bayesian estimation of the time-varying parameters model with stochastic volatility, as described below and in more detail in Appendix B (see steps 2 to 8).

More specifically, we assume that \(\tilde{y}_t\) can be written as:

\[
\tilde{y}_t = A_{0,t} + A_t(L)\tilde{y}_{t-1} + \varepsilon_t
\]  

(2)

where, \(A_{0,t}\) is the vector of time-varying intercepts, \(A_t(L) = A_{1,t}L + A_{2,t}L + ... + A_{l,t}L^{l-1}\) is a matrix polynomial of time-varying coefficients in the lag operator \(L\) and \(\varepsilon_t\) is the vector of innovations. Let \(\mathbf{A}_t = [A_{0,t}, A_{1,t}, ... A_{l,t}]\) and \(\theta_t = vec(\mathbf{A}_t)\), where \(vec(\cdot)\) is the column stacking operator. The law of motion for \(\theta_t\) is assumed to be such that:

\[
\theta_t = \theta_{t-1} + \omega_t,
\]

\(^3\)The algorithm can be applied to any mixed-frequency or irregularly spaced observations environment.

\(^4\)In our empirical application, the low-frequency is annual, the high frequency is quarterly, and τ is 1999.
where $\omega_t$ is a Gaussian white noise with zero mean and covariance $\Omega$.

The innovations in equation (2) are assumed to be Gaussian white noises with zero mean and time-varying covariance $\Sigma_t$ that is factorized as:

$$\Sigma_t = F_t D_t F_t',$$

where $F_t$ is lower triangular, with ones on the main diagonal, and $D_t$ is a diagonal matrix. Let $\sigma_t$ be the vector of the diagonal elements of $D_t^{1/2}$ and the off-diagonal element of the matrix $F_t^{-1}$. We assume that the standard deviations, $\sigma_t$, evolve as geometric random walks, belonging to the class of models known as stochastic volatility. The contemporaneous relationships $\phi_{it}$ in each equation of the VAR are assumed to evolve as an independent random walk, leading to the following specifications:

$$\log \sigma_t = \log \sigma_{t-1} + \zeta_t$$
$$\phi_{it} = \phi_{it-1} + \varphi_{it}$$

where $\zeta_t$ and $\varphi_{it}$ are Gaussian white noise with zero mean and covariance $\Xi$ and $\Psi_i$, respectively. We assume that $\epsilon_t, \omega_t, \zeta_t,$ and $\varphi_{it}$ are mutually uncorrelated at all leads and lags and that $\varphi_{it}$ is independent of $\varphi_{jt}$ for $i \neq j$.

### 2.2 Priors specification

In this Subsection, we discuss the specification of our priors. In particular, we make the following assumptions about the priors’ densities. First, the coefficients of the covariances of the log volatilities and the hyperparameters are assumed to be independent of each other. The priors for the initial states $\theta_0, \phi_0$ and $\log \sigma_0$ are assumed to be normally distributed. The priors for the hyperparameters, $\Omega, \Xi$ and $\Psi$ are assumed to be distributed as independent inverse-Wishart. More precisely, we have
the following prior specifications:

- **Time-varying coefficients:** \( P(\theta_0) = N(\hat{\theta}, \hat{V}_\theta) \) and \( P(\Omega) = IW(\Omega_0^{-1}, \rho_1) \);

- **Diagonal elements:** \( P(\log \sigma_0) = N(\log \hat{\sigma}, I_n) \) and \( P(\Psi_i) = IW(\Psi_{0i}^{-1}, \rho_{3i}) \);

- **Off-diagonal elements:** \( P(\phi_{i0}) = N(\hat{\phi}_i, \hat{V}_{\phi_i}) \) and \( P(\Xi) = IW(\Xi_0^{-1}, \rho_2) \);

where the scale matrices are parametrized as follows: \( \Omega_0^{-1} = \lambda_1 \rho_1 \hat{V}_\theta, \Psi_{0i} = \lambda_3 \rho_{3i} \hat{V}_{\phi_i} \) and \( \Xi_0 = \lambda_2 \rho_2 I_n \). The state vector \( \tilde{y}_t \) is initialized by linear interpolation while the hyper-parameters are initialized using a time invariant recursive VAR estimated on a sub-sample consisting of the first \( T_0 \) observations (see Subsection 3.2 for details on the empirical application). For the initial states, \( \theta_0 \), and the contemporaneous relations, \( \phi_{i0} \), we set the means, \( \hat{\theta} \) and \( \hat{\phi}_i \), and the variances, \( \hat{V}_\theta \) and \( \hat{V}_{\phi_i} \), as the maximum likelihood estimates (estimates of the variances are multiplied by four). For the initial states of the log volatilities, \( \log \sigma_0 \), the mean of the distribution is chosen to be the logarithm of the point estimates of the standard errors of the residuals of the estimated time invariant VAR. The degrees of freedom for the covariance matrix of the drifting coefficient’s innovations are set equal to \( T_0 \) the size of the initial-sample. The degrees of freedom for the priors on the covariance of the stochastic volatilities’ innovations, are set equal to the minimum necessary to ensure that the prior is proper. In particular, \( \rho_1 \) and \( \rho_2 \) are equal to the number of rows of \( \Xi_0^{-1} \) and \( \Psi_{0i}^{-1} \) plus one, respectively. The parameter \( \lambda_1 \) is fixed to 0.004, while \( \lambda_2 \) and \( \lambda_3 \) to 0.0001.\(^5\) Estimation is performed by discarding the explosive draws.

\(^5\)For the choice of \( \lambda_1 \), we follow the strategy developed by D’Agostino and Ehrmann (2014) based on the in-sample accuracy of the fitted data. Very loose values of \( \lambda_1 \) would imply a large variance of the distribution of the coefficients, and hence a large variance of the distribution of the fitted values. In this case, the model would tend to overfit the data, and an overly large percentage of observed data would lie within the confidence bands around the fitted values. The opposite would happen if \( \lambda_1 \) is very tight. Ideally, we would like to observe that 1% of the observed data lies outside the 1% confidence bands, 2% lies outside the 2% confidence bands and so on. We fix \( \lambda_1 \) as the value that minimizes the distance of the actual percentages from their theoretically expected values in the government spending equation. We focus on the choice of \( \lambda_1 \) which is very relevant because this is the parameter governing the tightness of the covariance matrix of the time-varying coefficients (results are robust to changes of \( \lambda_2 \) and \( \lambda_3 \)).
2.3 Related literature based on mixed-frequency models

The literature on models for variables sampled at different frequencies has evolved considerably recent years.\(^6\) Foroni and Marcellino (2013) propose an exhaustive survey of this literature, which mainly focuses on bridge equation models, MIXed DAta Sampling (MIDAS) models, mixed frequency VARs and mixed frequency factor models.\(^7\) Related to this work, Schorfheide and Dongho (2015) evaluate forecasts from a mixed-frequency VAR and compare them to a standard quarterly-frequency VAR and to forecasts from MIDAS regressions, based on a real-time dataset. Recently, D’Agostino et al. (2015) have developed a framework for measuring and monitoring business cycles in real time, building on a dynamic factor model which allows for heterogenous lead-lag patterns of the various indicators and mixed-frequency data. While an exhaustive overview of this literature is beyond the scope of the present paper, here we focus on some papers that, as in our work, add some time-varying features to their mixed-frequency framework. We also highlight some similarities and differences with respect to our approach.

Notably, Carriero et al. (2015) develop a Bayesian mixed-frequency model with stochastic volatility and time-varying parameters. Their approach implies transforming each time series of high-frequency (monthly) indicators into three series of low-frequency (quarterly) indicators, each containing observations for, respectively, the first, second or third month of the quarter. The method then consists of running a time-varying univariate regression on variables at the quarterly frequency.

Marcellino et al. (2015) propose a mixed-frequency dynamic factor model that allows for stochastic volatility for both the latent common factor and the idiosyncratic component of quarterly and monthly variables. Their model is an extension of Mariano and Murasawa (2003)’s model in a Bayesian environment, in which they relax the assumption of constant volatility. In this framework,

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\(^6\)See, e.g., Banbura and Modugno (2010), Ghysels (2012), Banbura et al. (2013).

\(^7\)See also Foroni et al. (2013) for a review focused on mixed-frequency VAR models. It is worth noting that - with the exception of a few papers on fiscal monitoring and forecasting (see, e.g., Camba-Mendez and Lamo (2004), Onorante et al. (2010), Asimakopoulos et al. (2013)) - little has been done so far in the field of fiscal policy analysis.
quarterly variables are defined as an average of the monthly factor. The latter is generated via the Koopman and Durbin (2003) smoother, which is more efficient in terms of computational speed than the Carter and Kohn (1994)’s smoother. The unobservable volatilities are generated via the algorithm developed by Jacquier et al. (1995). In our framework, we instead use the algorithm of Kim et al. (1998), which provides smaller autocorrelation of the draws and at the same time is more efficient.

A related strand of the literature focuses on MIDAS models. In this context, Schumacher (2014) proposes an extension to MIDAS with time-varying parameters. Galvao (2013) develops a smooth transition (ST-) MIDAS, in which a transition function shapes the change in some parameters of the MIDAS regression. In contrast to this approach, and similarly to our framework, Schumacher (2014) assumes that the time-varying parameters evolve according to a random walk and are therefore not linked to specific regimes.

All in all, to the best of our knowledge, the present paper is the first attempt to extend the Cogley and Sargent (2005) and Primiceri (2005) TV-BVAR methodology to allow for estimation on data sampled at different frequencies.

3 The effects of government spending shocks in Italy

We employ the TV-MF-BVAR methodology described in Section 2 to study the effects of government spending shocks in Italy over the period 1988Q4-2013Q3, thus including most of the recent crisis. In our view, the analysis of Italy is interesting, given that this country has witnessed many structural changes since the 1980s and has recently experienced a very severe recession. As discussed above, Italy is also a relevant case for the application of a mixed-frequency approach because the national account accrual time series for the main fiscal variables are available only beginning in 1999.8

8Most of the literature on fiscal policy has employed national accounts accrual data rather than cash data (see, e.g., Blanchard and Perotti (2002) and Mountford and Uhlig (2009)). In fact, while the latter are generally available
3.1 Data

Our benchmark VAR includes three variables: (i) government spending, which is computed as
government consumption plus government investment, (ii) GDP, (iii) the short-term nominal interest
rate.\footnote{See also Appendix A for a detailed description of the data and their sources. Note that, for this application, the
Gibbs sampling algorithm generally converges when no more than three variables are included in the VAR system.
This is due to its computational complexity, which adds one step to the Primiceri (2005)’s method, and the high
persistence of the variables. In this application, most of the draws with four variables are discarded because they are
unstable.} For the latter, we use the average interest rate on Italian government T-bills, i.e., government
securities with a maturity of one year or less.\footnote{The use of long-term interest rates does not lead to significantly different results in our analysis (see Section 3.4.}
In line with the reference literature (see, e.g., Perotti (2007)), we transform government spending and GDP into logs of real per capita terms by first
dividing the nominal series by the GDP deflator and, then, taking the ratio of the real series to
total population.\footnote{We deflate government spending using the GDP deflator because the government investment deflator is not
available on a quarterly basis from national account statistics for Italy.} Finally, we apply the natural logarithm. The interest rate is not transformed.
The data set covers the period 1981Q4-2013Q3 and includes quarterly observations for GDP and
the interest rate over this period. For government spending, annual data are available from 1981
until 1998, and quarterly data for the subsequent period 1999Q1-2013Q3. This represents the
mixed-frequency feature of our data set, as highlighted in Figure 1.\footnote{As commonly done in the related literature (see, e.g., Blanchard and Perotti (2002)), government spending is
constructed as the sum of government consumption and government investment. Quarterly data for government
investment are available only as of 1999Q1, whereas government consumption data are available in a non-seasonally
adjusted form as of 1991Q1. Given that the variable used in the VAR analysis is the sum of these two components,
we treat government spending as fully available at the quarterly frequency since 1999Q1.}

Including GDP allows us to analyse how the government spending multiplier - i.e., the percentage
change of GDP following a 1% of GDP shock to government spending - has evolved in Italy over a
period that includes the recent global crisis. At the same time, using an interest rate on sovereign

securities is important in order to capture the interdependencies between fiscal policies and the sovereign debt market as well as to address the following questions: (i) how does the interest rate on government securities react to an expansionary (or contractionary) fiscal shock and, vice versa, (ii) how does government spending respond to a shock to the interest rate on government securities? These issues are particularly relevant for Italy, which has seen a remarkable decline in sovereign yields in the run-up to the EMU, coupled with a tightening of government expenditure (see Figure 1). Interest rates on sovereign securities stabilized during the first phase of the EMU, while government spending rose again until the beginning of the recent crisis. Since 2008, Italy has experienced a new government spending contraction which was triggered by the consolidation policies adopted during the crisis. This was accompanied by a sharply declining GDP and, in some phases, rising interest rates on government securities.

Our baseline model is estimated on both annual data (for government spending prior to 1999) and quarterly data (for GDP, the interest rate, and government spending since 1999Q1). Annual and quarterly data for government consumption, investment and GDP are retrieved from the ECB’s Government Financial Statistics (GFS) and Eurostat. The latter validates the national account statistics produced by the Italian statistical agency (ISTAT) according to the ESA accounting standards. The data on interest rates on government securities are retrieved from the IMF’s International Financial Statistics dataset. The starting observation in our sample is 1981 for annual data (government spending) and 1981Q4 for quarterly data (GDP, interest rate). Thus, we end up with a dataset which comprises quarterly data for real GDP and the short-term interest rate for the period 1981Q4-2013Q3, quarterly data for government spending for the interval 1999Q1-2013Q3, and annual data for government spending for the period between 1981 and 1998. The methodology presented in Section 2 allows us to backcast the quarterly profile of the variable sampled at an annual frequency (government spending). The model produces such a backcast by exploiting the
cross-sectional covariation across all variables.

3.2 Starting conditions and generation of missing data

As discussed in Section 2, the estimation of the TVP-MF-VAR model requires initializing the state vector $\tilde{y}_t$ of missing observations and the hyper-parameters. The state vector is initialized by linear interpolation while the hyper-parameters are calibrated using a time invariant recursive VAR estimated using a sub-sample consisting of the first $T_0 = 28$ observations, i.e., from 1981Q4 until 1988Q3.

Figure 2 shows the quarterly government spending data for the available sample (1999Q1-2013Q3), together with the quarterly series generated by our model for the period 1988Q4-1998Q4. Grey bars represent the 68% confidence bands around the generated data. In addition, to test the ability of the model to generate missing observations, we perform a simulation exercise. We replace the true data of the government spending series for the first three quarters of each year, from 1999 until the end of the sample, with missing observations. We then generate the missing data through our model and plot the generated observations’ distribution against the true values. Figure 3 shows the results and indicates that the model accurately generates the true observations, which mainly fall into the confidence bands around the generated data points. We compute the MSE as the average squared distance between the median estimates and the true data and compare it to that obtained with a simple linear interpolation. The MSE are respectively 0.61 and 0.72 and suggest a superior accuracy from the TV-MF-VAR model. Finally, Figure 4 illustrates the fit of the model, using 68% confidence bands around the true data for the period 1999Q1-2013Q3. The model is shown to track the data in-sample well.

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13 That is, in our framework with low (annual) and high (quarterly) frequency observations for spending, missing high-frequency observations are computed as a linear interpolation of low-frequency data points.

14 The expectation and the covariance of the initial states, $E(\tilde{Y}_{0|0})$ and $\tilde{P}_{0|0}$, are initialized with the unconditional mean and the identity matrix, respectively.

15 To get consistent figures between annual and quarterly data, annual data points before 1999Q1 are divided by four prior to the estimation.
3.3 Results

Figures 5 and 6 show the main results of the paper in terms of impulse response functions to the government spending shock. Following Fatás and Mihov (2001) and Blanchard and Perotti (2002), the government spending shock is identified assuming that government spending is predetermined in a system including other macroeconomic variables, in this case output and the interest rate.\textsuperscript{16}

Specifically, Figure 5 plots the quarter-specific impulse response of government spending (first chart), GDP (second chart) and the interest rate (third chart) to a government spending shock equal to 1% of GDP over the 1988Q4-2013Q3 sample. As shown in Figure 5, the impulse response of government spending to its own shock is broadly stable over the entire sample, for all quarters. Conversely, the GDP response to the spending shock appears to vary remarkably over time. In particular, GDP reacts strongly in the first part of the sample, from 1988 until the beginning of the 1990s: in this period, the multiplier peaks at around 1.5 at short horizons, i.e., up to one year after the shock. Then, the short-term GDP reaction declines below unity in the run-up phase to the EMU and until the recent crisis. During the crisis, we observe a further increase in the short-term GDP multiplier, which reaches peak values above one. At the same time, the average short-term multiplier for the crisis period is around one. Over the full sample, the longer-term impulse response of GDP to the spending shock tends to decline towards zero, indicating that the real effects of government spending shocks are short-lived and vanish after around one year.

Overall, Figure 5 suggests that spending shocks have stronger effects on output during slowdowns, as reflected in the output response peaking during both the early 1990s recession and in the recent recession beginning in the late 2000s. This might be due to the presence of a higher number of credit-constrained agents in these phases of the business cycle (Gál et al. (2007)), possibly coupled

\textsuperscript{16}The literature has proposed alternative identification schemes. In Section 3.4, we propose several robustness exercises, including two different identification approaches. See also Caldara and Kamps (2008) and Mertens and Ravn (2010) for a broader discussion on identification issues in fiscal policy analysis.
With respect to the interest rate response to the spending shock, a declining pattern emerges from the third chart of Figure 5. The interest rate response appears to be much stronger in the first part of the sample, coinciding with the early 1990s and run-up phase to the single currency, compared with the EMU period. This might reflect greater financial market prudence with respect to the sustainability of Italian public finances in the 1990s compared to the 2000s. As a consequence, Italian securities likely carried a higher risk premium following expansive fiscal policies during this period. By the same token, if one analyses the mirror image of Figure 5, i.e., the effects of negative spending shocks, it can be argued that the consolidation policies put in place in Italy in the 1990s with a view to meeting the Maastricht criteria and joining the single currency were particularly effective in reducing interest rates on sovereign securities. During the EMU period, the interest rate response appears quite muted. The crisis period is characterized by a somewhat more unstable interest rate reaction to the spending shock, which stem from a more erratic behaviour of investors in sovereign securities during this period, compared with the pre-crisis period.

Figure 6 shows the impulse responses of GDP (first column) and the interest rate (second column) to the government spending shock, together with 68% confidence bands, in three selected quarters: 1988Q4, 1999Q1, 2011Q1. The first quarter is the initial quarter in our sample and the second corresponds to the start of the EMU, while 2011Q1 is the central quarter in the recent crisis period, which in our sample covers the period 2008Q3-2013Q3.\(^{18}\)

This analysis shows that the GDP response tends to be statistically significant in the short-term, up to six quarters following the shock for 1988Q4. For longer horizons, the GDP impulse responses are generally not significant, thus indicating that government spending shocks lose power

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\(^{17}\)The U-shaped GDP multiplier seem to be driven primarily by the private consumption component of GDP, rather than its private investment component (results not shown, available from the authors). See also Kirchner et al. (2010), Beetsma and Giuliodori (2011), Corsetti et al. (2012) and Coenen et al. (2012) for a more encompassing discussion on the factors that may affect the government spending multiplier.

\(^{18}\)Results are virtually unchanged when we consider the last quarter in the sample (2013Q3), instead of 2011Q1.
to stimulate output after around one year. The interest rate response to the spending shock is positive and statistically significant in the short-term for 1988Q4 and 1999Q1, but not significant for 2013Q3. In particular, Figure 6 suggests that, in 1988Q4, a 1% of GDP spending shock triggers a 130 basis point increase in the interest rate after one year. Such effect is weaker in 1999Q1, at only around 80 basis points.\textsuperscript{19}

### 3.4 Robustness analysis

This Section presents several robustness exercises for the empirical application of the paper described above.

**Long-term interest rate and tighter prior assumptions.** The upper chart in Figure 7 reports the impulse responses of GDP to the 1% of GDP spending shock throughout the 1988Q4-2013Q3 sample when the 10-year interest rate on Italian government securities is included in the baseline VAR, replacing the short-term interest rate used in the baseline specification. The lower chart of Figure 7 shows the impulse response of GDP from the baseline model (including government spending, GDP and the short-term interest rate) where tighter assumptions for the priors are used.\textsuperscript{20}

In both cases, the GDP response pattern to the spending shock is confirmed: the spending multiplier appears to peak at values above unity at the beginning of the sample, coinciding with the late 1980s and the early 1990s. In both cases, the GDP response reaches its highest levels about one year after the shock. Then, the multiplier tends to decline to a value below 1 by the mid-1990s. The multiplier then stabilizes at the 0.7-0.9 interval before rising again in the context of the recent crisis, when it reaches around 1.4.

\textsuperscript{19}Our results show that the instability of the GDP multiplier, as reflected in statistically significant differences in the GDP impulse response to the spending shock between two different quarters, is only limited to particular periods. However, it should be noted that the variance of our estimates is larger than that obtained with a standard time-varying VAR, e.g., à la Primiceri (2005). In fact, our estimation steps involve an additional draw for sampling the unknown observations that are treated as unobservable states, which increases the variance of the estimates.

\textsuperscript{20}In particular, the tightness parameter $\lambda_1$ is fixed to 0.002. This more stringent specification compared to the baseline model, where $\lambda_1 = 0.004$, implies less time variation in the coefficients.
**Net taxes.** We have estimated an alternative VAR system in which we have included net taxes as a first variable in the VAR model, together with government spending and GDP as in Blanchard and Perotti (2002). We follow this ordering for the identification based on the Cholesky recursive scheme. As in Blanchard and Perotti (2002), net taxes are defined as government revenues less transfers and are transformed in real and per capita terms. Revenues are available on a quarterly basis from the ECB’s government finance statistics beginning in 1999Q1, and on an annual basis before 1999. Therefore, this exercise includes two variables characterized by mixed-frequency data: government spending and net taxes. Figure 8 shows the impulse responses to a net tax shock (first column) and government spending shock (second column). The tax shock is shown to exert a slightly positive spending response in the short run, while spending declines in the long-run. The GDP response is negative over the whole horizon except on impact. Taxes appear to react positively to the spending shock. The GDP response to the spending shock shows a U-shaped pattern which is very similar to the baseline exercise, with a short-term impact that is especially strong in the latter part of the sample.

**Alternative identification schemes.** We perform two robustness exercises to test whether our main results on the GDP multipliers are robust to the identification choice adopted. Our baseline identification implies that government spending is pre-determined with respect to GDP and the interest rate, following the assumption that governments do not react immediately to unexpected movements in GDP and the interest rate. In the first exercise, we have swapped the ordering of government spending and GDP, thus imposing that GDP cannot react contemporaneously to spending shocks. The first column of Figure 9 shows the effects of the government spending shock on the other two variables, supporting the paper’s main findings, given that the response of GDP follows a U-shape over the sample considered, while the interest rate response exhibits a declining pattern over time. As a second robustness exercise on identification, we have identified the government
spending shock using sign restrictions, in the spirit of Mountford and Uhlig (2009). Our identifying assumptions are broadly based on Mountford and Uhlig (2009) and Caldara and Kamps (2008): using the algorithm proposed in Rubio-Ramirez et al. (2010), we impose that the government spending shock generates a positive response of government spending on impact. We also identify a business cycle shock and an interest rate shock. We assume that, on impact: the business cycle shock induces a positive response of GDP, the interest rate shock triggers a interest rate positive response and a negative GDP response. In addition, we impose orthogonality among the three shocks. The second column of Figure 9 - where, for simplicity, we only focus on the responses to the spending shock - plots the impulse responses of the government spending shock identified through this approach. Again, it emerges that the GDP response is higher in the early and final parts of the sample, while the interest rate response is stronger only in the first part of the sample.

**Industrial production and employment.** The two charts reported in Figure 10 show the impulse response of industrial production (total industry excluding construction) and number of employees (total economy) to the government spending shock. To estimate these impulse responses, we run the baseline VAR model but we replace GDP with the industrial production index and then with employment. The industrial production index is available for the full sample at a monthly frequency. We therefore transform the data to a quarterly frequency before incorporating them into the VAR model. The employment series is already released at a quarterly frequency for the full sample and we do not therefore implement any transformation. For both variables, we take the natural logarithm - as we did for GDP - such that we can interpret the impulse responses as percentage changes following the 1% of GDP government spending shock.

It emerges that the industrial production reaction follows a U-shape, much like the GDP response. However, the industrial production response is even stronger at the start of the sample, with the “IP multiplier” reaching values close to 2 until the early 1990s, and then declining to around
zero in the following periods. The short-term industrial production response to the fiscal shock picks up again during the recent crisis when the multiplier once again exceeds unity for horizons below one year. The employment response to the spending shock is less volatile (Figure 10, lower chart). This is not surprising given that the employment series is characterized by a much higher persistence compared to industrial production or GDP. Still, a pattern qualitatively similar to the one of GDP and industrial production emerges: the employment response reaches around 0.8 at the start of the sample, then declines to around 0.7 between the mid-1990s and the late 2000s, before once again rising to around one during the crisis.\footnote{Our baseline results are also robust to the inclusion of the debt-to-GDP ratio in the model (results not shown, available from the authors).}

Overall, these results support the findings from the baseline model. In particular, the real effects of spending shocks seem to peak in the late 1980s and early 1990s, declining in the subsequent period and stabilizing during the first phase of the EMU. Then, such effects become stronger again starting from the late 2000s, in the recent crisis.

3.5 Related literature and comparison with our results

The empirical research on the effects of fiscal policy shocks has developed relatively rapidly in recent years, especially since the seminal work by Blanchard and Perotti (2002) on the United States. However, studies on European countries are scarce, mainly due to limited availability of long quarterly fiscal time series compiled on an accrual basis. In fact, with the exception of a few countries (e.g., Germany), before 1999, most European national statistical offices did not produce comprehensive quarterly time series of fiscal data in accordance with internationally agreed accounting standards (e.g., ESA1995 or ESA2010 accounting rules).

One of the first papers to estimate the effects of fiscal shocks in European countries is Marcellino (2006). Based on semi-annual data from the OECD, which has now been discontinued, this paper...
estimates the effects of fiscal shocks in Germany (and three other EU countries) over the period 1981-2001. It is shown that spending shocks have weak effects on German output tax shock effects are more sizeable and of the expected (negative) sign.

A Bayesian time-varying model for the analysis of fiscal shocks has been proposed by Kirchner et al. (2010). This paper focuses on the aggregate euro area, based on the quarterly fiscal data set compiled by Paredes et al. (2009). The results show that, for the period 1980-2008, the short-run effectiveness of government spending in stimulating real GDP and private consumption increased until the end-1980s but decreased thereafter. The paper also highlights that rising government debt is the main reason for declining spending multipliers at longer horizons.

With respect to Italy, there are very few studies on the effects of fiscal shocks. One exception is Giordano et al. (2007). The authors construct a quarterly cash data set for selected fiscal variables over the period 1982-2004, mainly relying on the information contained in the Italian Treasury Quarterly Reports. The paper suggests that a one percent government spending shock increases private real GDP by 0.6 per cent after three quarters. This response fades away after two years. More recently, Caprioli and Momigliano (2011) propose new estimates of expenditure and revenue shocks for Italy. They find that fiscal shocks tend to have significant effects on economic activity. These effects appear to be stronger, as well as more precisely estimated and robust, for expenditure shocks.

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22 Pereira and Lopes (2010) is another paper that uses a time-varying Bayesian VAR model for the analysis of fiscal shocks. Based on a U.S. dataset for the period 1965-2009, the paper suggests that fiscal policy lost some capacity to stimulate output over time. Other studies explore the changing effects of fiscal shocks based on subsamples or rolling windows of data. For example, Beetsma and Giuliodori (2011) show that, in the EU, the output effects of government spending shocks tend to be lower in the pre-EMU period than in the EMU period. Based on rolling-window estimates, Cimadomo and Bénassy-Quéré (2012) find that the net tax multiplier follows a humped-shaped curve in Germany, peaking in the middle of the 1990s. Government spending shocks are shown to be more powerful to stimulate output after the German reunification.

23 A related set of papers focuses on the “state-dependent” effects of fiscal policies, in particular on whether fiscal policies are more or less effective in stimulating output in different phases of the business cycle. In this context, some studies have found that fiscal multipliers in advanced economies tend to be higher in downturns, and especially when the zero lower bound on the nominal interest rate binds (Christiano et al. (2011)), than in expansions (see, e.g., Auerbach and Gorodnichenko (2012), Baum et al. (2012), Batini et al. (2012)). Unlike these papers, Owyang et al. (2013) suggest that, when considering a longer time period (i.e., from 1890 to 2010), there is no strong evidence that multipliers are higher during recessions in the United States.
Our results also indicate that the effects of fiscal shocks are maximized in the short-run, i.e., until around the one-year horizon. However, unlike these papers on Italy, we find a higher spending multiplier (on average, at around one, with peaks at around 1.5). Apart from the differences in the underlying samples, these discrepancies suggest that the use of accrual vs. cash data tend to lead to different results as regards the effects of fiscal shocks.

4 Conclusions

This paper has two main contributions. First, we add to the mixed-frequency literature by proposing a time-varying parameter VAR model with stochastic volatility which makes it possible to use arbitrary data patterns of mixed-frequency or irregularly spaced observations. While literature on mixed-frequency models has developed remarkably over recent years, we propose an approach aimed at extending the Cogley and Sargent (2005) and Primiceri (2005)’s time-varying BVAR model to a mixed-frequency setting. The proposed methodology allows for the study of the effects of structural shocks in an environment characterized by short time series of high-frequency data, and to adapt it to cases of regime switches and parameter instability.

Second, we apply this model to study the effects of fiscal policy in Italy, which have remained largely unexplored due to the unavailability of long time series of quarterly accrual fiscal data. We investigate how the transmission of government spending shocks has changed over time for Italy, which has a vulnerable fiscal position and has spent long periods at the centre of financial market tensions during the recent crisis. Based on a sample incorporating both annual and quarterly variables for the 1988Q4-2013Q3 period, our results indicate that, in Italy, the fiscal multiplier tends to follow a U-shape. It reaches around 1.5 at short-term horizons and at the beginning of the 1990s, and then stabilizes at the 0.8-0.9 interval during the run-up phase to the EMU and until the start of the recent crisis, before rising again to above unity during the recent recession. The
average short-term multiplier is around one during the recent crisis, i.e., in the period from 2008Q3 (Lehman crisis) until 2013Q3 (end of our sample).

Finally, we show that the reaction of the interest rate on Italian short-term government securities to the spending shock was stronger in the 1990s than in the EMU period. This may indicate that the Italian spending cuts implemented in the 1990s in order to meet the Maastricht criteria and join the single currency were particularly effective in reducing interest rates on sovereign securities.
References


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Figure 1: Variables in the baseline VAR: government spending, i.e., government consumption plus investment expenditure, GDP and the short-term nominal interest rate. The latter is the average interest rate on Italian government T-bills, i.e., government securities with maturity up to one year. Government spending and GDP are in real per capita terms, and are transformed taking logs. The interested rate is in levels. Sample: 1981Q4-2013Q3.

Figure 2: Quarterly government spending included in the baseline VAR. For the period 1999Q1-2013Q3, quarterly data are available from Eurostat or the ECB’s Government Financial Statistics (green circles). For the period before 1999Q1, only annual data are available for this variable. Annual data before 1999Q1 are divided by four to make them comparable with the quarterly data. The missing quarterly data before 1999Q1 are generated based on the model described in Section 2. The back diamonds are the median of the distribution of generated data, bars represent 68% confidence bands around the generated data.
Figure 3: True and generated observations over the sample 1999Q1-2013Q3. The green circles represent the true data, the back diamonds are the median of the distribution of generated data, fan charts are the 95%, 90%, 85% and 80% confidence bands around the generated data.

Figure 4: Model fit over the sample 1999Q1-2013Q3. Stars are the true data, solid lines the 68% confidence bands around the data generated by the model.
Figure 5: Impulse response of government spending (first chart), GDP (second chart) and the interest rate (third chart) to a government spending shock equal to 1% of GDP. Sample: 1988Q4-2013Q3.

Figure 6: Impulse responses of GDP (first column) and the interest rate (second column) to a government spending shock equal to 1% of GDP in three selected quarters (1988Q4, 1999Q1, 2011Q1), together with 68% confidence bands.
Figure 7: GDP response to a government spending shock equal to 1% of GDP generated by a VAR model that includes the 10-years government bond yield, replacing the short-term interest rate (upper chart) and with different prior assumptions (lower chart), as specified in section 3.4 (lower chart). Sample: 1988Q4-2013Q3.

Figure 8: VAR model including net taxes, government spending and GDP. First column: impulse responses of net taxes, spending and GDP to a 1% of GDP net tax shock. Second column: impulse response of taxes, spending and GDP to a 1% of GDP government spending shock. Sample: 1988Q4-2013Q3.
Figure 9: First column: impulse responses of GDP, government spending and the interest rate to a government spending shock, where the VAR ordering is GDP, G and IR and the spending shock is identified based on a Cholesky factorization. Second column: the government spending shock is identified based on sign restrictions. Sample: 1988Q4-2013Q3.

Figure 10: Response of industrial production (upper chart) and employment, i.e., number of employees (lower chart), to a government spending shock equal to 1% of GDP. Sample: 1988Q4-2013Q3.
Appendix A: Data

This Appendix describes the data used in the paper and their sources.

- **Government spending**: this series is constructed as the sum of general government consumption plus general government investment. Quarterly data for government investment are available only as of 1999Q1, whereas government consumption data are available in a non-seasonally adjusted form as of 1991Q1. Given that the variable used in the VAR analysis is the sum of these two components, we treat government spending as fully available at the quarterly frequency as of 1999Q1. The quarterly data are retrieved from the ECB’s Government Financial Statistics (GFS). The annual data, covering the period 1980 until 1998, are retrieved from the European Commission’s AMECO database.

- **Net taxes**: this series is constructed as government revenues minus transfers. Both series are available from the ECB’s Government Financial Statistics (GFS) at the quarterly frequency as of 1999Q1, whereas at the annual frequency before 1999.

- **Real Gross Domestic Product (GDP)**: this series is published at the quarterly frequency by the Italian statistical agency (ISTAT). It is available in seasonally and working days adjusted terms and as of 1981.

- **Short-term interest rate**: we use the average interest rate on Italian government T-bills, i.e., government securities with maturity equal or less than one year. The series is retrieved from the IMF’s International Financial Statistics (IFS). Data are available at the monthly frequency as of March 1977. We transform the series at the quarterly frequency prior to estimation.

- **Long-term interest rate**: we use the 10-year government bond yield. The series is retrieved from the Bank of International Settlements (BIS) dataset. Data are available at the monthly
frequency as of January 1960. We transform the series at the quarterly frequency prior to estimation.

- GDP deflator: we use the quarterly GDP deflator index, which is published at the quarterly frequency in the IMF’s IFS. Data are available as of 1980Q1.

- Population (total): this series is used in order to obtain per capita values for the relevant variables. The series is retrieved from Eurostat’s ESA95 National Account dataset and it is available at the quarterly frequency as of 1992Q1, and at the annual frequency from 1980 onwards. For the period from 1980 until 1991, we generate quarterly series assuming that population remain constant over the four quarters in a specific year.

- Industrial production index: we use the Italian Industrial Production Index (Total Industry) published by the ECB. Data are available at the monthly frequency - working day and seasonally adjusted - as of January 1980. We therefore transform the series at the quarterly frequency.

- Employment: we use total employment data (excluding construction) which is published at the quarterly frequency by Eurostat, in its ESA95 National Accounts. Data are available as of 1980Q1.

**Appendix B: The Bayesian algorithm**

This Appendix describes in detail the estimation of the mixed-frequency model outlined in Section 2 of the paper.

Estimation is done using Bayesian methods. To draw from the joint posterior distribution of model parameters we use a Gibbs sampling algorithm. The basic idea of the algorithm is to draw sets of coefficients from known conditional posterior distributions. The algorithm is initialized at
some values and, under some regularity conditions, the draws converge to a draw from the joint posterior after a burn in period. Let \( z \) be \((q \times 1)\) vector, we denote \( z^T \) the sequence \([z_1', ..., z_T']\).

Each repetition is composed of the following steps:

1. \( p(\tilde{y}^T|y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi, s^T) \)

2. \( p(s^T|y^T, \tilde{y}^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi)^{24} \)

3. \( p(\phi^T|y^T, \tilde{y}^T, \theta^T, \sigma^T, \Omega, \Xi, \Psi, s^T) \)

4. \( p(\theta^T|y^T, \tilde{y}^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi, s^T) \)

5. \( p(\Omega|y^T, \tilde{y}^T, \theta^T, \sigma^T, \phi^T, \Xi, \Psi, s^T) \)

6. \( p(\Xi|y^T, \tilde{y}^T, \theta^T, \sigma^T, \phi^T, \Omega, \Psi, s^T) \)

7. \( p(\Psi|y^T, \tilde{y}^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, s^T) \)

Gibbs sampling algorithm

- Step 1: sample from \( p(\tilde{y}^T|y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi, s^T) \)

  Draws for \( \tilde{y}_t \) can be obtained from a \( N(\tilde{y}_t|\tilde{y}_{t+1}, \tilde{P}_{t+1}) \), where \( \tilde{y}_{t+1} = E(\tilde{y}_t|\tilde{y}_{t+1}, y^T, \theta^T, \sigma^T, \Omega, \Xi, \Psi, s^T) \) and \( \tilde{P}_{t+1} = var(\tilde{y}_t|\tilde{y}_{t+1}, y^T, \theta^T, \sigma^T, \Omega, \Xi, \Psi, s^T) \) are obtained with the algorithm of Carter and Kohn (1994).

- Step 2: sample from \( p(s^T|y^T, \tilde{y}^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi) \)

  Conditional on \( y^*_{i,t} \) and \( r^T \), we independently sample each \( s_{i,t} \) from the discrete density defined by \( Pr(s_{i,t} = j|y^*_{i,t}, r_{i,t}) \propto q_j f_N(y^*_{i,t} + 2r_{i,t} - 1.2704, v_j^2) \), where \( f_N(y|\mu, \sigma^2) \) denotes a normal density with mean \( \mu \) and variance \( \sigma^2 \); \( q_j, m_j \) and \( v_j \) are chosen to match the moment of the \( log(\chi^2) \) distribution.

\(^{24}\)See below the definition of \( s^T \).
• Step 3: sample from \(p(\sigma^T | y^T, \tilde{y}^T, \theta^T, \phi^T, \Omega, \Xi, \Psi, s^T)\)

To draw \(\sigma^T\) we use the algorithm of Kim et al. (1998) (hereafter KSC). Consider the system of equations \(\tilde{y}_t^* \equiv F_t^{-1}(y_t - X_t' \theta_t) = D_t^{1/2} u_t\), where \(u_t \sim N(0, I)\), \(X_t = (I_n \otimes x_t')\), and \(x_t = [1_n, \tilde{y}_{t-1}...\tilde{y}_{t-p}]\). Conditional on \(\tilde{y}^T, \theta^T, \phi^T, \tilde{y}^*_t\) is observable. Squaring and taking the logarithm, we obtain

\[
\tilde{y}_t^{**} = 2r_t + \nu_t
\]

\[
r_t = r_{t-1} + \xi_t
\]

where \(\tilde{y}_t^{**} = \log((\tilde{y}_t^*)^2 + 0.001)\) - the constant (0.001) is added to make estimation more robust - \(\nu_{i,t} = \log(u_{i,t}^2)\) and \(r_t = \log \sigma_{i,t}\). Since, the innovation in (3) is distributed as \(\log \chi^2(1)\), we use, following KSC, a mixture of 7 normal densities with component probabilities \(q_j\), means \(m_j - 1.2704\), and variances \(v_j^2\) \((j=1,...,7)\) to transform the system in a Gaussian one, where \(\{q_j, m_j, v_j^2\}\) are chosen to match the moments of the \(\log \chi^2(1)\) distribution. The values are:

<table>
<thead>
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<th>(j)</th>
<th>(q_j)</th>
<th>(m_j)</th>
<th>(v_j^2)</th>
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<td>6.0000</td>
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</tr>
<tr>
<td>7.0000</td>
<td>0.2575</td>
<td>-1.0882</td>
<td>1.2626</td>
</tr>
</tbody>
</table>

Let \(s^T = [s_1,...,s_T]'\) be a matrix of indicators selecting the member of the mixture to be used for each element of \(\nu_t\) at each point in time. Conditional on \(s^T\), \((\nu_{i,t}|s_{i,t} = j) \sim N(m_j - 1.2704, v_j^2)\). Therefore we can use the algorithm of Carter and Kohn (1994) to draw \(r_t\) \((t=1,...,T)\) from \(N(r_t|t+1, R_t|t+1)\), where \(r_t|t+1 = E(r_t|t+1, y^T, \tilde{y}^T, \theta^T, \phi^T, \Omega, \Xi, \Psi, s^T)\) and \(R_t|t+1 = Var(r_t|t+1, y^T, \tilde{y}^T, \theta^T, \phi^T, \Omega, \Xi, \Psi, s^T)\).
• Step 4: sample from $p(\phi^T | y^T, \tilde{y}^T, \theta^T, \sigma^T, \Omega, \Xi, \Psi, s^T)$

Consider again the system of equations $F_t^{-1}(\tilde{y}_t - X_t^T \theta_t) = D_t^{1/2} u_t$. Conditional on $\theta^T$, $\tilde{y}_t$ is observable. Since $F_t^{-1}$ is lower triangular with ones in the main diagonal, each equation in the above system can be written as

$$\hat{y}_{1,t} = \sigma_{1,t} u_{1,t} \quad (5)$$

$$\hat{y}_{i,t} = -\hat{y}_{[1,i-1],t} \phi_{i,t} + \sigma_{i,t} u_{i,t} \quad i = 2, ..., n \quad (6)$$

where $\sigma_{i,t}$ and $u_{i,t}$ are the $i$th elements of $\sigma_t$ and $u_t$ respectively, $\hat{y}_{[1,i-1],t} = [\hat{y}_{1,t}, ..., \hat{y}_{i-1,t}]$. Under the block diagonality of $\Psi$, the algorithm of Carter and Kohn (1994) can be applied equation by equation, obtaining draws for $\phi_{i,t}$ from a $N(\phi_{i,t} | \phi_{i,t+1}, y^t, \tilde{y}^t, \theta^T, \sigma^T, \Omega, \Xi, \Psi)$ and $\Phi_{i,t,t+1} = Var(\phi_{i,t} | \phi_{i,t+1}, y^t, \tilde{y}^t, \theta^T, \sigma^T, \Omega, \Xi, \Psi)$.

• Step 5: sample from $p(\theta^T | y^T, \tilde{y}^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi, s^T)$

Conditional on all other parameters and the observables we have

$$\bar{y}_t = X_t^T \theta_t + \varepsilon_t \quad (7)$$

$$\theta_t = \theta_{t-1} + \omega_t \quad (8)$$

Draws for $\theta_t$ can be obtained from a $N(\theta_{t|t+1}, P_{t|t+1})$, where $\theta_{t|t+1} = E(\theta_t | \theta_{t+1}, y^t, \tilde{y}^t, \sigma^T, \phi^T, \Omega, \Xi, \Psi)$ and $P_{t|t+1} = Var(\theta_t | \theta_{t+1}, y^t, \tilde{y}^t, \sigma^T, \phi^T, \Omega, \Xi, \Psi)$ from the algorithm of Carter and Kohn (1994).

• Step 6: sample from $p(\Omega | y^T, \tilde{y}^T, \theta^T, \sigma^T, \phi^T, \Xi, \Psi, s^T)$

Conditional on the other coefficients and the data, $\Omega$ has an Inverse-Wishart posterior density with scale matrix $\Omega_1^{-1} = (\Omega_0 + \sum_{t=1}^{T} \Delta \theta_t (\Delta \theta_t)^T)^{-1}$ and degrees of freedom $df_{\Omega_1} = df_{\Omega_0} + T$, where
\( \Omega_0^{-1} \) is the prior scale matrix, \( df_{\Omega_0} \) are the prior degrees of freedom and \( T \) is length of the sample use for estimation. To draw a realization for \( \Omega \) make \( df_{\Omega_1} \) independent draws \( z_i \) (\( i=1,\ldots,df_{\Omega_1} \)) from \( N(0,\Omega_1^{-1}) \) and compute \( \Omega = (\sum_{i=1}^{df_{\Omega_1}} z_i z_i')^{-1} \) (see Gelman et. al., 1995).

- Step 7: sample from \( p(\Xi|y^T, \tilde{y}^T, \theta^T, \sigma^T, \phi^T, \Omega, \Psi, s^T) \)

Conditional the other coefficients and the data, \( \Xi \) has an Inverse-Wishart posterior density with scale matrix \( \Xi_1^{-1} = (\Xi_0 + \sum_{t=1}^{T} \Delta \log \sigma_t (\Delta \log \sigma_t)^{-1} \) and degrees of freedom \( df_{\Xi_1} = df_{\Xi_0} + T \) where \( \Xi_0^{-1} \) is the prior scale matrix and \( df_{\Xi_0} \) the prior degrees of freedom. Draws are obtained as in step 5.

- Step 8: sample from \( p(\Psi|y^T, \tilde{y}^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, s^T) \).

Conditional on the other coefficients and the data, \( \Psi_i \) has an Inverse-Wishart posterior density with scale matrix \( \Psi_{i,1}^{-1} = (\Psi_{i,0} + \sum_{t=1}^{T} \Delta \phi_{i,t} (\Delta \phi_{i,t})^{-1} \) and degrees of freedom \( df_{\Psi_{i,1}} = df_{\Psi_{i,0}} + T \) where \( \Psi_{i,0}^{-1} \) is the prior scale matrix and \( df_{\Psi_{i,0}} \) the prior degrees of freedom. Draws are obtained as in step 5 for all \( i \). The estimations are performed with 12000 repetitions discarding the first 10000 and collecting one out of five draws.

Based on the estimation procedure outlined above, Figure 4 shows the in-sample fit of the model for the empirical application illustrated in Section 2 of the paper. The model fit is reported for the sample period 1999Q1-2013Q3. The straight lines refer to the median fit and its 68% confidence bands, stars refer to the realised observations. The model seems to track well the observed dynamics.