Risk Management Optimization for Sovereign Debt Financing

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European Stability Mechanism

ESM, Luxembourg, Dec. 2018.

Some history

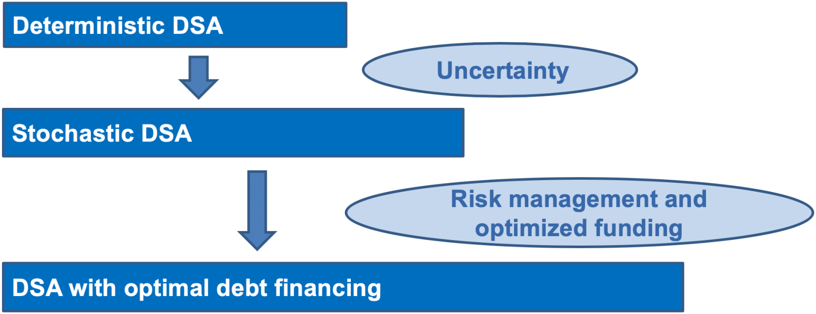
- Consiglio and Zenios (January 2015)

  The devil is in the tails, `//Voxeu.org`.

- Consiglio and Zenios (2016)

  Risk management optimization for sovereign debt restructuring, *Journal of Globalization and Development*, 6(2):181–213, J. Stiglitz et al. (editors).

Research issues



Contributions

- Q1 Represent uncertainty
- Q2 Risk measurement
- Q3 Risk optimization of debt financing

1. Scenario trees
2. Introduce a risk measure
3. Simultaneously model debt stock and flow
4. Endogenous feedback of debt stock on flow
5. Optimize tradeoffs risk vs cost, and stock vs flow

Q2. Risk management for debt financing

- Risk management has not been part of analysis
- Wright (2012), Harvard Business Law Review:

  Need for development of criteria for "optimal" debt restructuring process.

- Missing normative models capturing complex tradeoffs
- Account for the reaction of the PDMO
- Optimization normalizes the test of forecasting modules

Q2. Risk management for debt financing

IMF Fiscal Monitor Report (2018)

The Wealth of Nations: Governments Can Better Manage What They Own and Owe

The economic problem

Sovereign output Y_t, primary balance PB_t, debt stock D_{t-1}

Model the optimal choice of debt financing variables X_t

The economic problem

- Flow dynamics

$$GFN_t = i_{t-1}D_{t-1} + A_t - PB_t$$

- Debt financing equation

$$\sum_{j=1}^{J} X_t(j) = GFN_t$$

- Endogenous premia $r_t(j) = r_{ft} + \rho(d_t, j)$
- Effective Interest Rate

$$i_t = \frac{i_{t-1}(D_{t-1} - A_t) + \sum_{j=1}^{J} r_t(j)X_t(j)}{D_t}$$

The economic problem

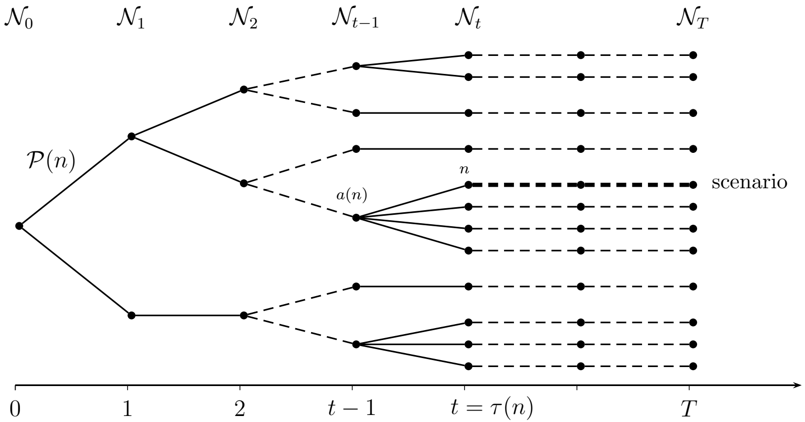
- Feedback loop

$$X \to D \to r \to X$$

- Uncertain correlated Y_t, PB_t, r_{ft}

Q1. Modeling uncertainty

- **First innovation: Scenario tree**



- Compact moment matching representation
- Discrete state- and time-space

Q1. Modeling uncertainty

- **First innovation: debt stock scenario dynamics**

$$D^n = (1 + r^{a(n)})D^{a(n)} - PB^n\ (+SF^n)$$

- Scenario dependent GDP

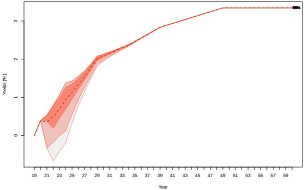
$$\begin{aligned} d^n &= D^n / Y^n \\ gfn_t^n &= GFN_t^n / Y_t^n \\ pb^n &= PB^n / Y^n \end{aligned}$$

- D^n is term structure of debt
- r^n is term structure of sovereign rates

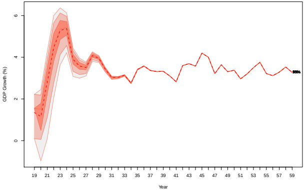
Scenario tree integrates economic and financial risk factors, using objective and risk neutral probabilities.

(Consiglio, Carollo, Zenios, *Quantitative Finance*, 16:201-212, 2016.)

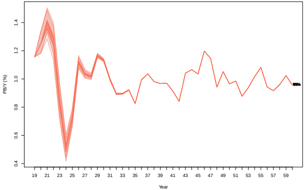
Q1. Modeling uncertainty



(a) Risk-free rates



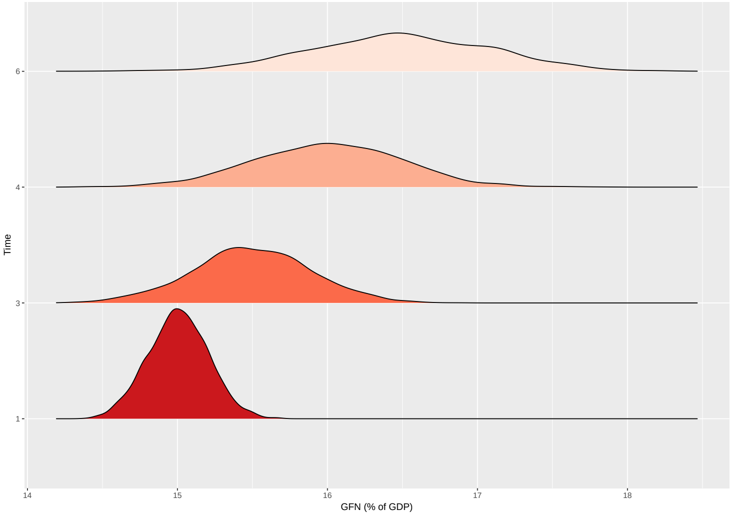
(b) GDP growth



(c) Primary balance

Q2. Risk measurement

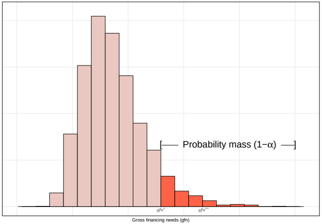
- Scenario dynamics of debt



Q2. Risk measurement

Second innovation: Conditional Flow at Risk (CFaR)

$$\Psi(gfn) \doteq \mathbb{E}\left(gfn \mid gfn \geq gfn^{\diamond}\right)$$



Rockafellar and Uryasev (2000,2002)

$$\Psi(gfn) = gfn^{\diamond} \quad + \quad \frac{1}{1-\alpha} \sum_{n \in \mathcal{N}} p^n z^n$$

$$z^n \quad \geq \quad gfn_t^n - gfn^{\diamond},\ z^n \geq 0$$

Q3. Risk optimization of debt financing

- Sovereign issues debt $X^n(j)$ to finance its debt
- $NIP_t^n = I_t^n + \sum_{m\in\mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m(j) CF_t^n(j,m)$
  (NIP/D is the effective interest rate of debt)
- Model (partial)

$$\begin{array}{ll} \mathrm{Minimize}_X & \sum_{n\in\mathcal{N}} p^n NIP_t^n \\ \text{s.t.} & \\ & \Psi(gfn) \ \leq \ \omega \end{array}$$

Q3. Risk optimization of debt financing

- What about *debt stock* dynamics?

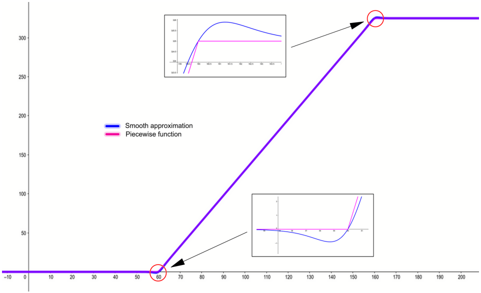
$$D_t^n = D_{t-1}^{a(n)} + GFN_t^n - \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} X_{\tau(m)}^m(j) \mathbb{1}^n(j,m) - A_t^n$$

Q3. Risk optimization of debt financingg

Third innovation: Endogeneity of interest rates

$$r_t^n(j) = r_{ft}^n + \rho(d_t^n, j)$$

$$\rho(d_t^n, j) = a_j + (1 + b_j)\hat{\rho}(d_t^n).$$



Q3. Risk optimization of debt financing

- **Model:** Optimize policy design

–Cost and risk tradeoffs
–Stock and flow tradeoffs
–Sustainability

$$\text{Minimize}_x \quad \sum_{n \in \mathcal{N}} p^n NIP_t^n$$

s.t.

$$\begin{aligned} \Psi(gfn) &\leq \omega \\ \frac{\partial d^n}{\partial t} &\leq \delta \end{aligned}$$

– Gross financing needs bounded by ω
– Pace of debt decrease $\delta < 0$ or debt increase $\delta > 0$
– Thresholds

Q3. Risk optimization of debt financing

1. **Conservative**
   Constrain the ratio for all states of the economy, i.e.,
   $$gfn^n \leq \omega, \text{ for all } n \in \mathcal{N}_t,\ t \in \mathcal{T}$$
2. **Risk neutral**
   Constrain the expected value of the ratio, i.e.,
   $$\mathbb{E}[gfn^n \mid n \in \mathcal{N}_t] \leq \omega, \text{for all } t \in \mathcal{T}$$
3. **Risk adjusted CFaR constrained**

Q3. Risk optimization of debt financing

To recap

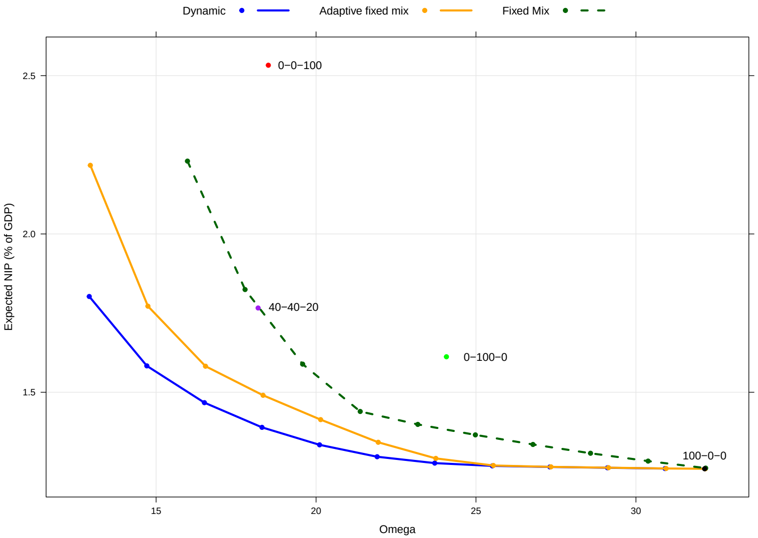
1. Scenario dynamics for both *debt stock* and *flow*
2. Risk measure
3. Interest rate endogeneity
4. Dynamic debt financing decisions

Climb-down

- Dynamic mix (time and state dependent)
- Adaptive fixed mix (time dependent, state invariant)
- Simple fixed mix rules (time and state invariant)

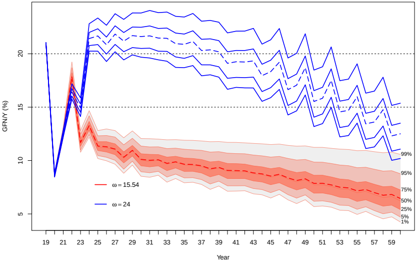
Q3. Risk optimization of debt financing

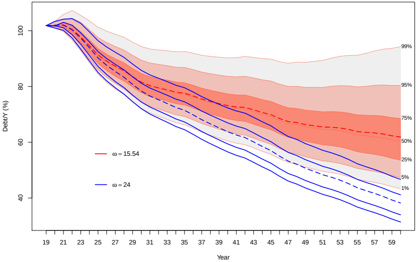
The relevance of optimizing



Q3. Risk optimization of debt financing

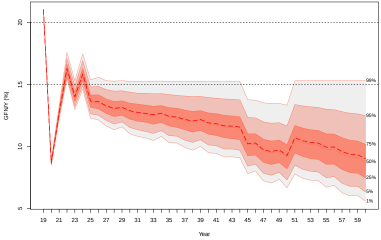
Tradeoff debt stock and debt flow

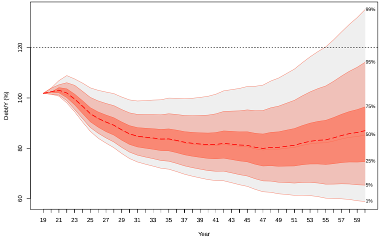




Q3. Risk optimization of debt financing

Additional fiscal effort and debt sustainability





$$\sum_{j=1}^{J} X_t^n(j) + u_t Y_t^n \geq GFN_t^n.$$

Q3. Risk optimization of debt financing

- Lesson 1. Risk management comes with a cost
- Lesson 2. Trading off debt flow and stock dynamics
- Lesson 3. Trade-offs are economically significant
- Lesson 4. Cost savings from optimization increase as risk tolerance declines
- Lesson 5. Optimizing renders less volatile financing needs but weighs on debt stock dynamics
- Lesson 6. Optimizing helps more when the stock of legacy debt is larger and its maturity shorter
- Lesson 7. Feedback from debt stock into interest rates affects risk management

Conclusions

- Rich framework for studying sovereign debt sustainability
  - Stochastic debt stock and flow dynamics
  - Coherent risk measure
  - Endogenous interest rates
  - Optimal fiscal stance
- Capture and quantify complex tradeoffs
- Replicate stylized model from economic literature: gambling for redemption (Conesa and Kehoe), cost of delays (Blanchard)
- Extension of feedback loop

$$X \to D \to r \to \mathbf{Y} \to \mathbf{PB} \to X$$

Publications

Athanasopoulou et al., *Risk management for sovereign financing within a debt sustainability framework*, European Stability Mechanism, Working Paper Series 31, Luxembourg, July 2018.

Consiglio, A. and S.A. Zenios, Risk management optimization for sovereign debt restructuring, *Journal of Globalization and Development*, 6(2):181–213, J. Stiglitz et al. (editors), 2016.

Consiglio, A., Carollo, A. and S.A. Zenios, A parsimonious model for multi-factor arbitrage-free scenario trees, *Quantitative Finance*, 16:201-212, 2016.