Risk Management Optimization for Sovereign Debt Financing

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European Stability Mechanism

Some history

- Consiglio and Zenios (January 2015)
  The devil is in the tails, //Voxeu.org.

- Consiglio and Zenios (2016)
Research issues

- Deterministic DSA
- Stochastic DSA

Uncertainty

Risk management and optimized funding

DSA with optimal debt financing
Contributions

Q1 Represent uncertainty
Q2 Risk measurement
Q3 Risk optimization of debt financing

1 Scenario trees
2 Introduce a risk measure
3 Simultaneously model debt stock and flow
4 Endogenous feedback of debt stock on flow
5 Optimize tradeoffs risk vs cost, and stock vs flow
Q2. Risk management for debt financing

- Risk management has not been part of analysis.

- Missing normative models capturing complex tradeoffs.
- Account for the reaction of the PDMO.
- Optimization normalizes the test of forecasting modules.
Q2. Risk management for debt financing


*The Wealth of Nations: Governments Can Better Manage What They Own and Owe*
Sovereign output $Y_t$, primary balance $PB_t$, debt stock $D_{t-1}$

Model the optimal choice of debt financing variables $X_t$
The economic problem

- **Flow dynamics**

\[ GFN_t = i_{t-1}D_{t-1} + A_t - PB_t \]

- **Debt financing equation**

\[ \sum_{j=1}^{J} X_t(j) = GFN_t \]

- **Endogenous premia**

\[ r_t(j) = r_{ft} + \rho(d_t, j) \]

- **Effective Interest Rate**

\[ i_t = \frac{i_{t-1}(D_{t-1} - A_t) + \sum_{j=1}^{J} r_t(j)X_t(j)}{D_t} \]
The economic problem

- Feedback loop

  \[ X \rightarrow D \rightarrow r \rightarrow X \]

- Uncertain correlated \( Y_t, PB_t, r_{ft} \)
Q1. Modeling uncertainty

- **First innovation: Scenario tree**

- **Compact moment matching representation**
- **Discrete state- and time-space**
Q1. Modeling uncertainty

- **First innovation: debt stock scenario dynamics**
  \[ D^n = (1 + r^{a(n)})D^{a(n)} - PB^n (+SF^n) \]

- Scenario dependent GDP
  \[ d^n = D^n/Y^n \]
  \[ gfn_t^n = GFN_t^n/Y_t^n \]
  \[ pb^n = PB^n/Y^n \]

- \(D^n\) is term structure of debt
- \(r^n\) is term structure of sovereign rates

Scenario tree integrates economic and financial risk factors, using objective and risk neutral probabilities.
(Consiglio, Carollo, Zenios, *Quantitative Finance*, 16:201-212, 2016.)
Q1. Modeling uncertainty

(a) Risk-free rates

(b) GDP growth

(c) Primary balance
Q2. Risk measurement

- Scenario dynamics of debt
Q2. Risk measurement

Second innovation: Conditional Flow at Risk (CFaR)

\[
\Psi(gfn) = \mathbb{E}(gfn \mid gfn \geq gfn^\circ)
\]

Rockafellar and Uryasev (2000, 2002)

\[
\Psi(gfn) = gfn^\circ + \frac{1}{1 - \alpha} \sum_{n \in \mathcal{N}} p^n z^n
\]

\[z^n \geq gfn_t^n - gfn^\circ, \quad z^n \geq 0\]
Q3. Risk optimization of debt financing

- Sovereign issues debt $X^n(j)$ to finance its debt

\[ NIP^n_t = l^n_t + \sum_{m \in P(n)} \sum_{j=1}^{J} X^m_{\tau(m)}(j) CF^n_t(j, m) \]

($NIP/D$ is the effective interest rate of debt)

- Model (partial)

\[ \text{Minimize}_{X} \sum_{n \in N} p^n NIP^n_t \]

s.t.

\[ \Psi(gfn) \leq \omega \]
What about \textit{debt stock} dynamics?

\[ D^n_t = D^{a(n)}_{t-1} + GFN^n_t - \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} X^m_{\tau(m)}(j) \mathbb{1}^n(j, m) - A^n_t \]
Q3. Risk optimization of debt financing

Third innovation: Endogeneity of interest rates

\[ r^n_t(j) = r^n_{ft} + \rho(d^n_t, j) \]

\[ \rho(d^n_t, j) = a_j + (1 + b_j)\hat{\rho}(d^n_t). \]
Q3. Risk optimization of debt financing

**Model:** Optimize policy design

- Cost and risk tradeoffs
- Stock and flow tradeoffs
- Sustainability

Minimize

$$\sum_{n \in N} p^n NIP^n$$

s.t.

$$\Psi(gfn) \leq \omega$$

$$\frac{\partial d^n}{\partial t} \leq \delta$$

- Gross financing needs bounded by $\omega$
- Pace of debt decrease $\delta < 0$ or debt increase $\delta > 0$
- Thresholds
Q3. Risk optimization of debt financing

1. **Conservative**
   Constrain the ratio for all states of the economy, i.e.,
   \[ gfn^n \leq \omega, \text{ for all } n \in N_t, \ t \in T \]

2. **Risk neutral**
   Constrain the expected value of the ratio, i.e.,
   \[ E[gfn^n \mid n \in N_t] \leq \omega, \text{ for all } t \in T \]

3. **Risk adjusted CFaR constrained**

Q3. Risk optimization of debt financing

To recap

1. Scenario dynamics for both *debt stock* and *flow*
2. Risk measure
3. Interest rate endogeneity
4. Dynamic debt financing decisions

Climb-down

- Dynamic mix (time and state dependent)
- Adaptive fixed mix (time dependent, state invariant)
- Simple fixed mix rules (time and state invariant)
Q3. Risk optimization of debt financing

The relevance of optimizing

![Graph showing risk optimization of debt financing](image)

- **Dynamic**
- **Adaptive fixed mix**
- **Fixed Mix**
Q3. Risk optimization of debt financing

Tradeoff debt stock and debt flow
Q3. Risk optimization of debt financing

Additional fiscal effort and debt sustainability

\[
\sum_{j=1}^{J} X_t^n(j) + u_t Y_t^n \geq GFN_t^n.
\]
Q3. Risk optimization of debt financing

- Lesson 1. Risk management comes with a cost
- Lesson 2. Trading off debt flow and stock dynamics
- Lesson 3. Trade-offs are economically significant
- Lesson 4. Cost savings from optimization increase as risk tolerance declines
- Lesson 5. Optimizing renders less volatile financing needs but weighs on debt stock dynamics
- Lesson 6. Optimizing helps more when the stock of legacy debt is larger and its maturity shorter
- Lesson 7. Feedback from debt stock into interest rates affects risk management
Conclusions

- Rich framework for studying sovereign debt sustainability
  - Stochastic debt stock and flow dynamics
  - Coherent risk measure
  - Endogenous interest rates
  - Optimal fiscal stance
- Capture and quantify complex tradeoffs
- Replicate stylized model from economic literature: gambling for redemption (Conesa and Kehoe), cost of delays (Blanchard)
- Extension of feedback loop
  \[ X \rightarrow D \rightarrow r \rightarrow Y \rightarrow PB \rightarrow X \]
